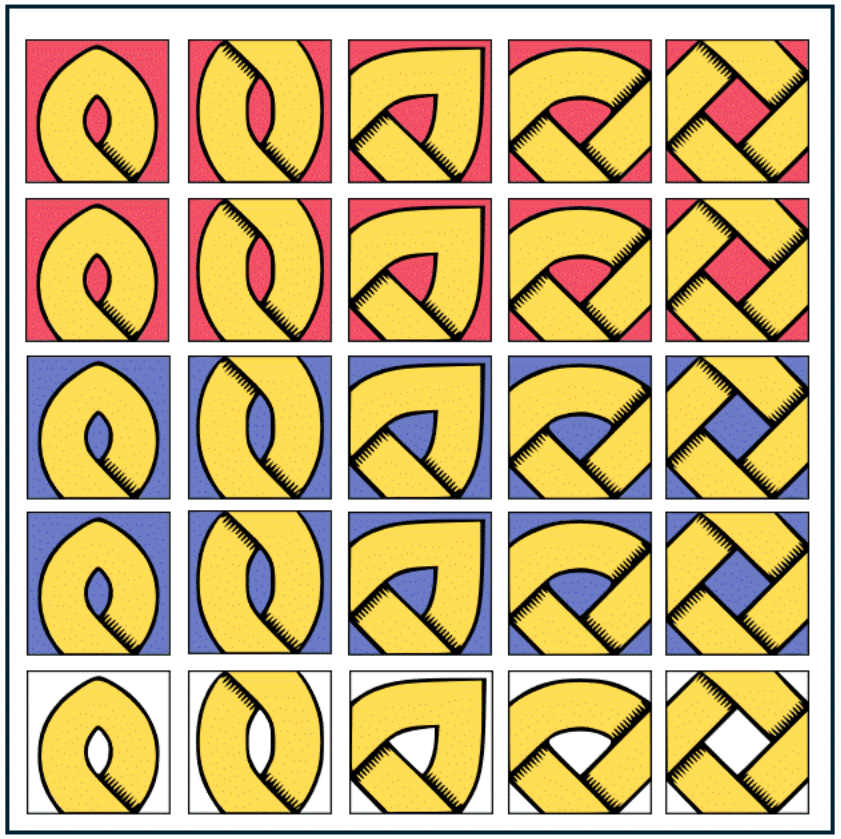
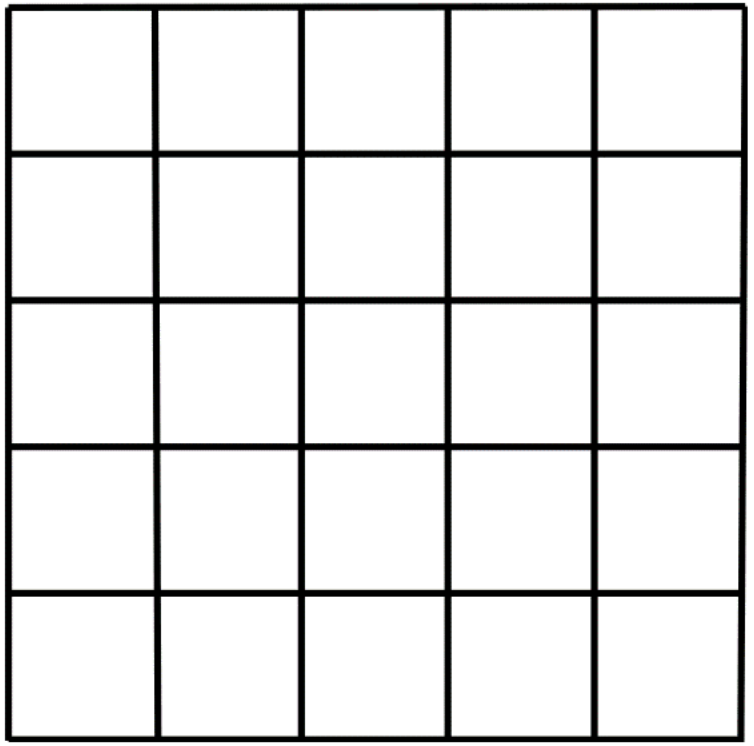
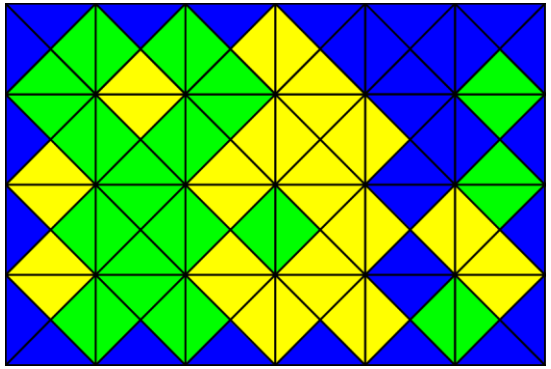
Tile-based Knot Assembly with Celtic!





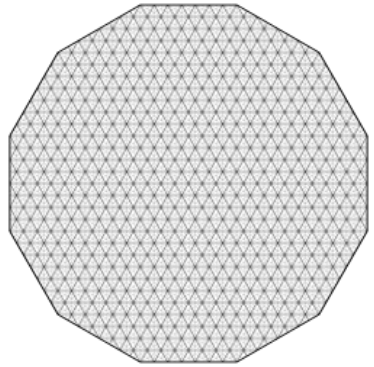
Authors: Divya Bajaj, **Ryan Knobel**, Juan Manuel Perez, Rene Reyes, Ramiro Santos, Tim Wylie

Background

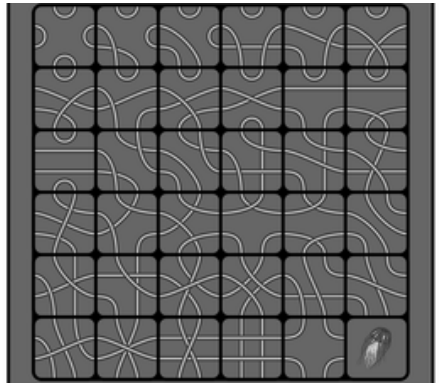


MacMahon Squares

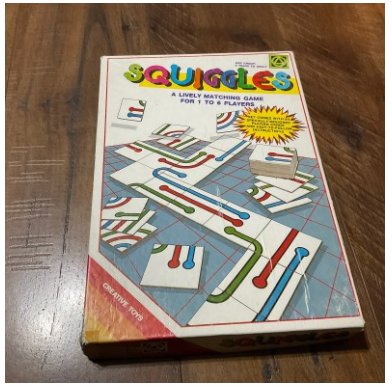
Jigsaw Puzzles



Eternity

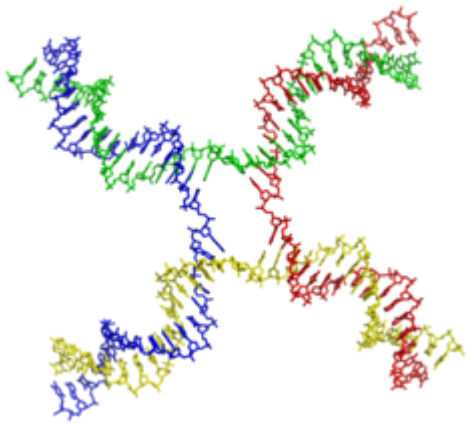


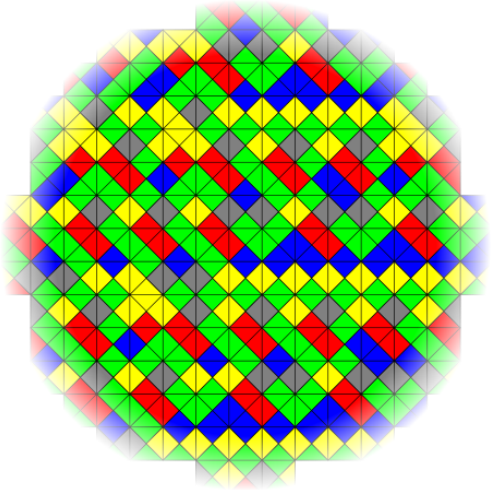
Tsuro

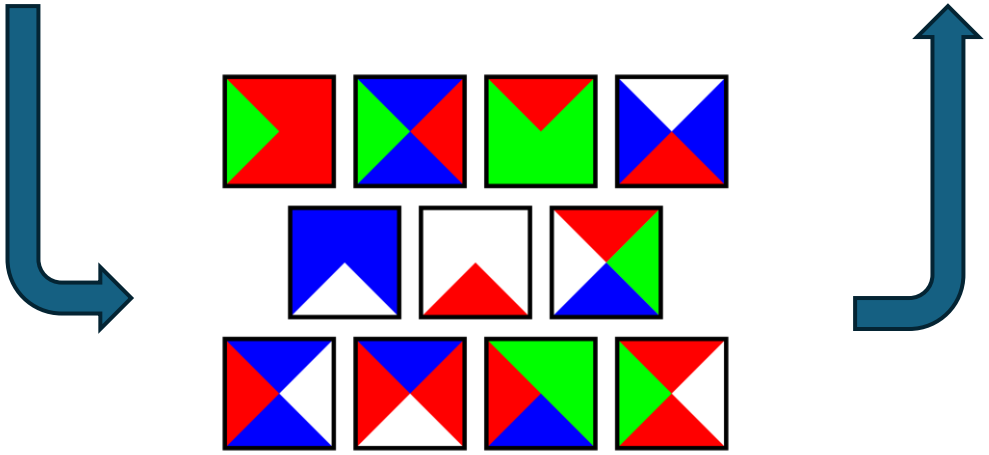


Squiggles

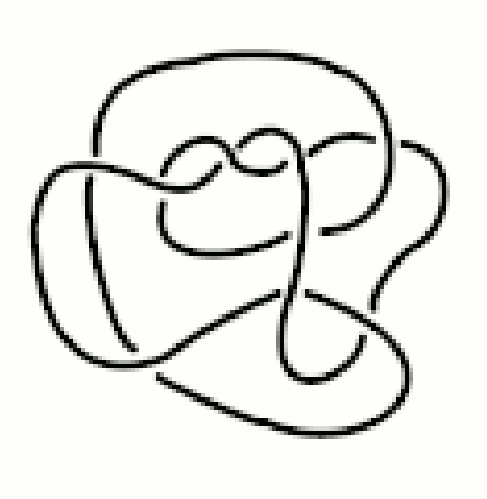
Background





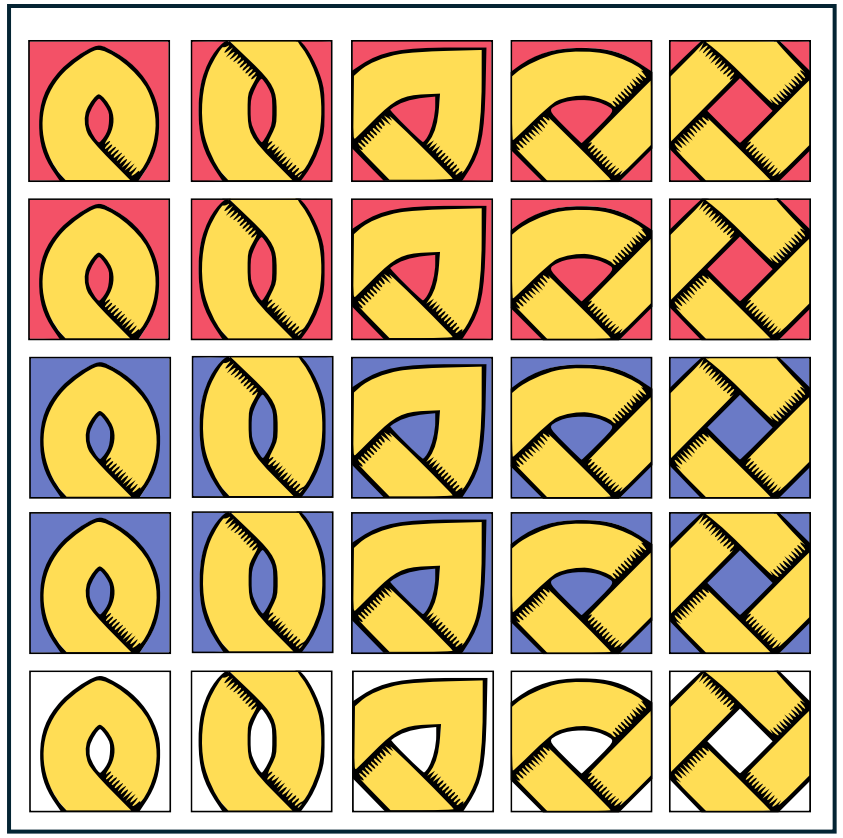


Wang tiles

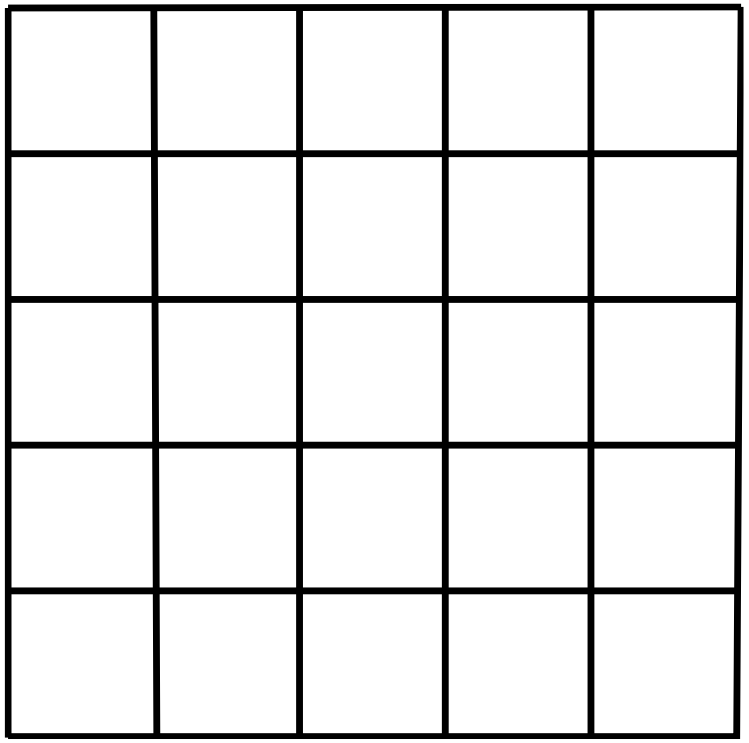


[Knot Theory Wikipedia]

Celtic!

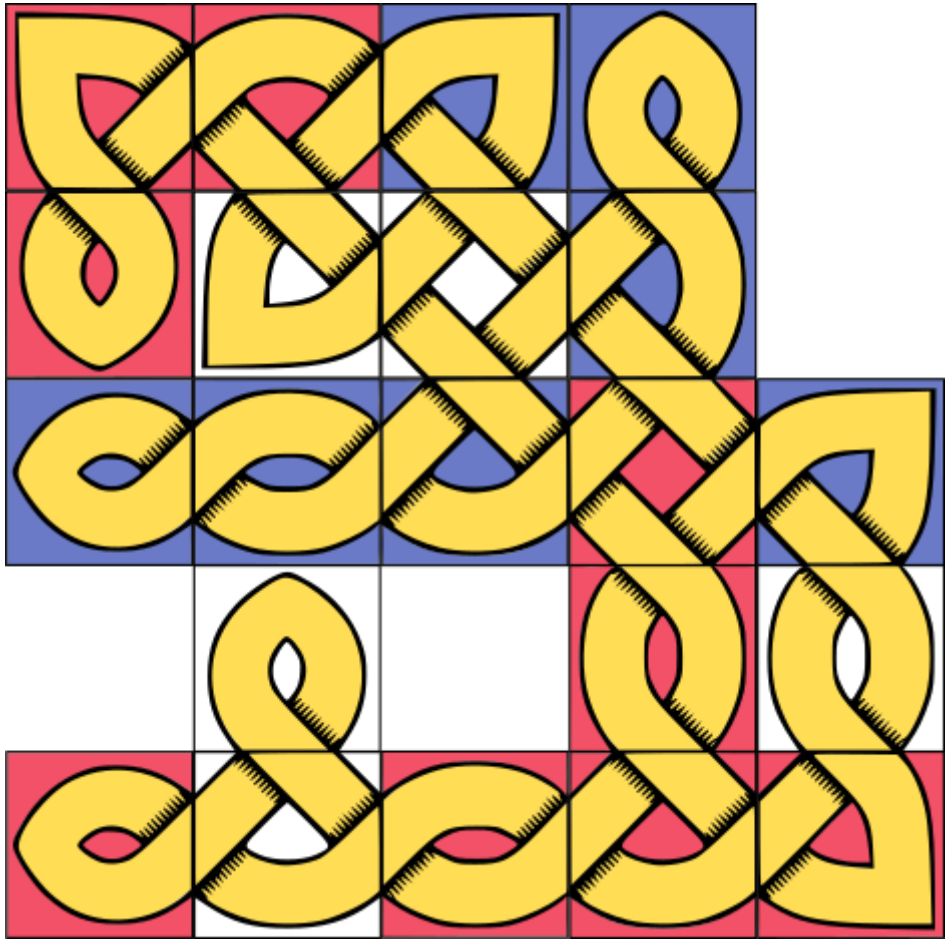


Pieces

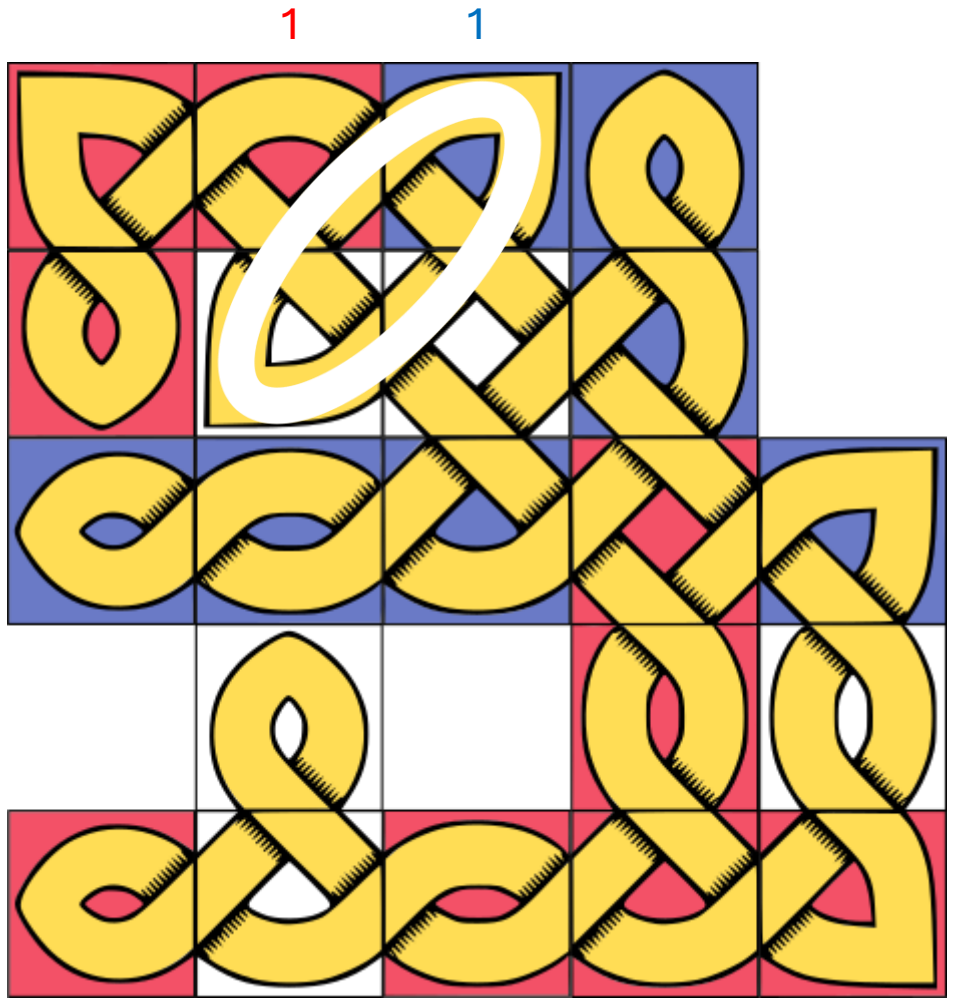


Board

Celtic! - Scoring



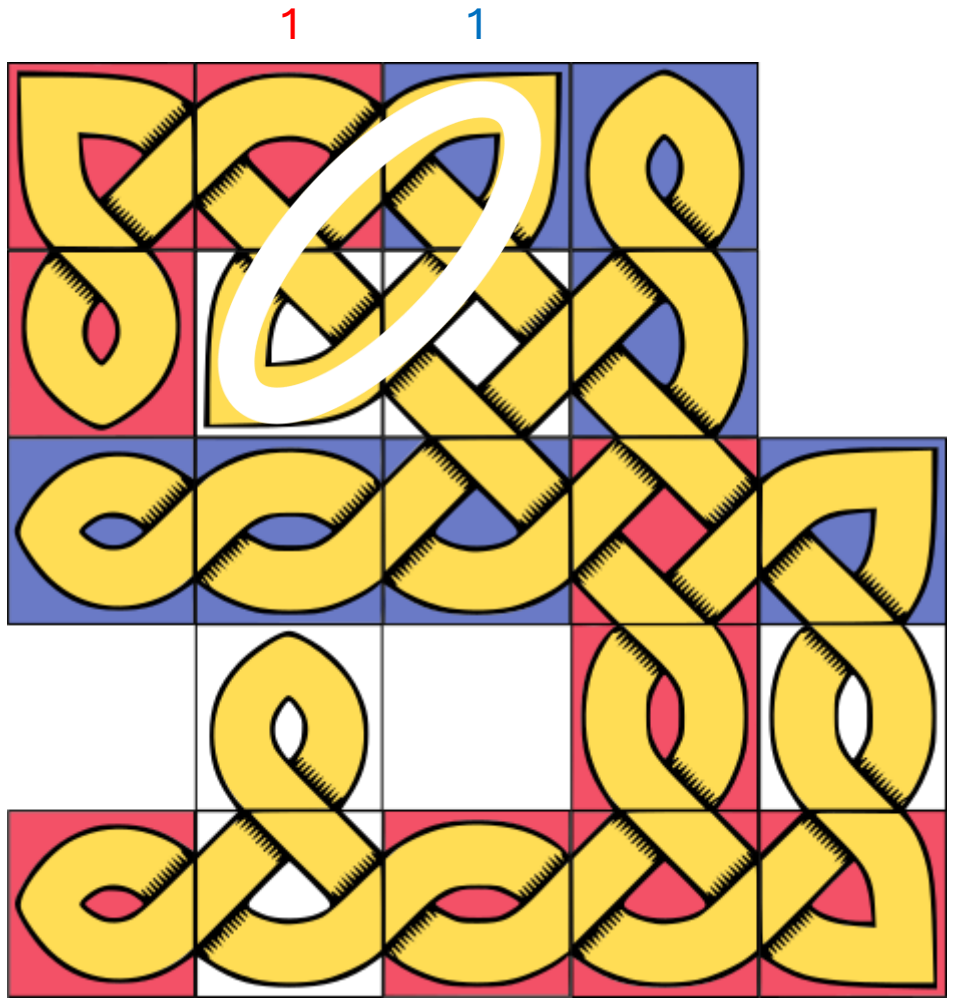
Celtic! - Scoring



Celtic! - Scoring

Score

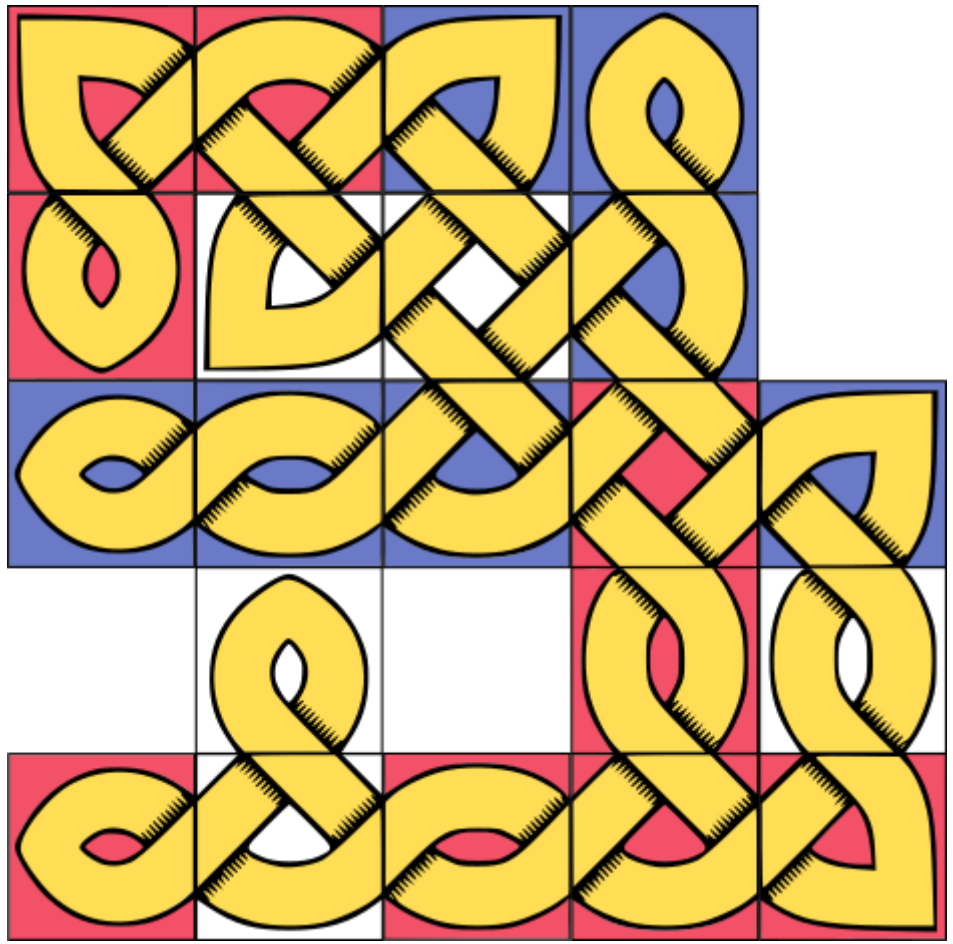
Knot 1: Tie



Celtic! - Scoring

Score

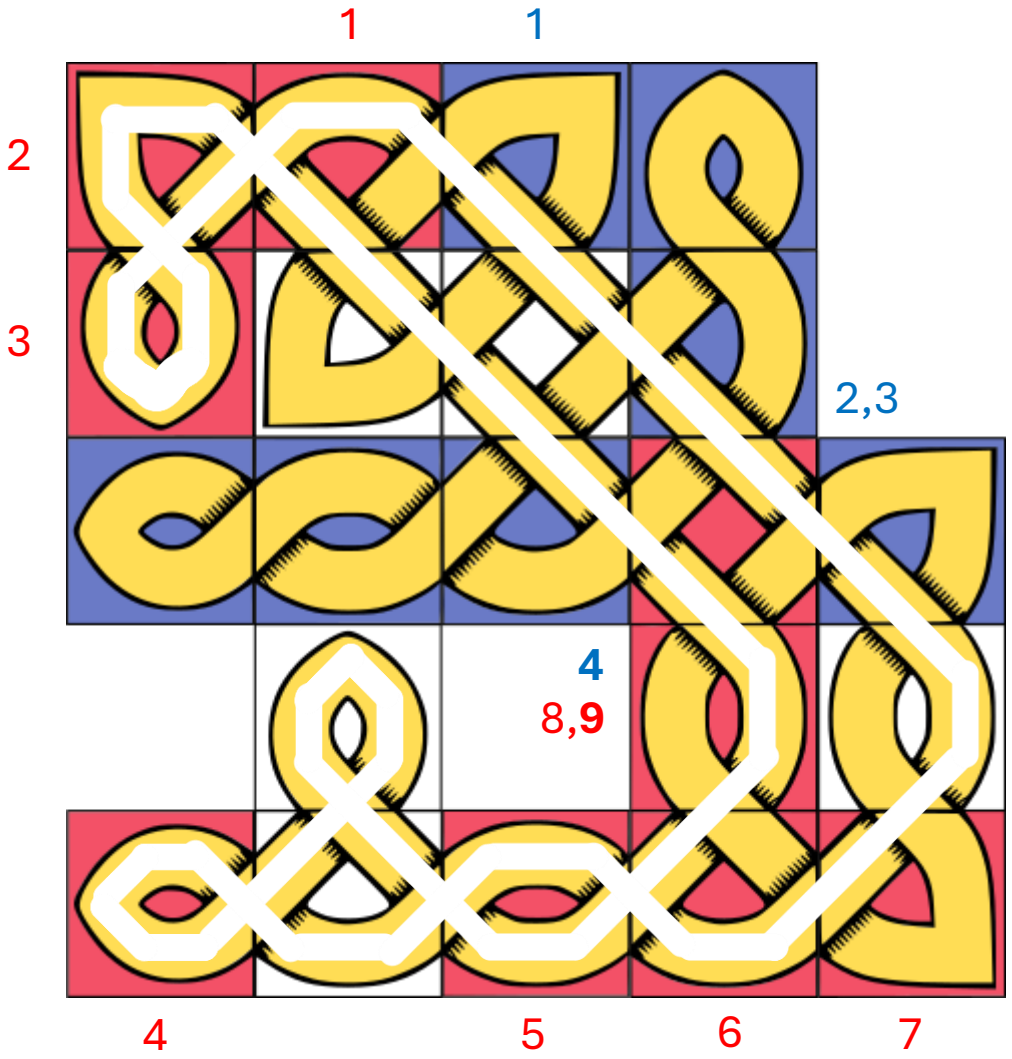
Knot 1: Tie



Celtic! - Scoring

Score

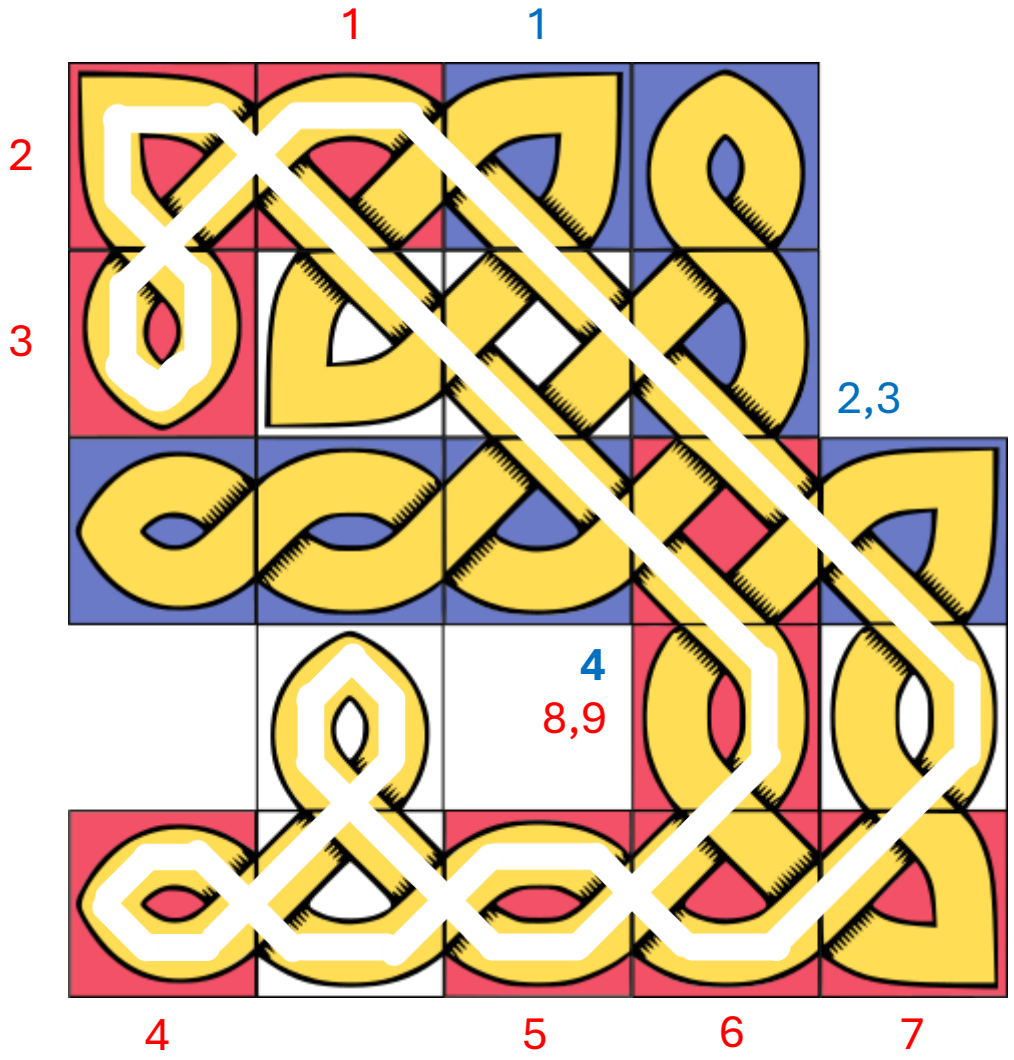
Knot 1: Tie



Celtic! - Scoring

Score

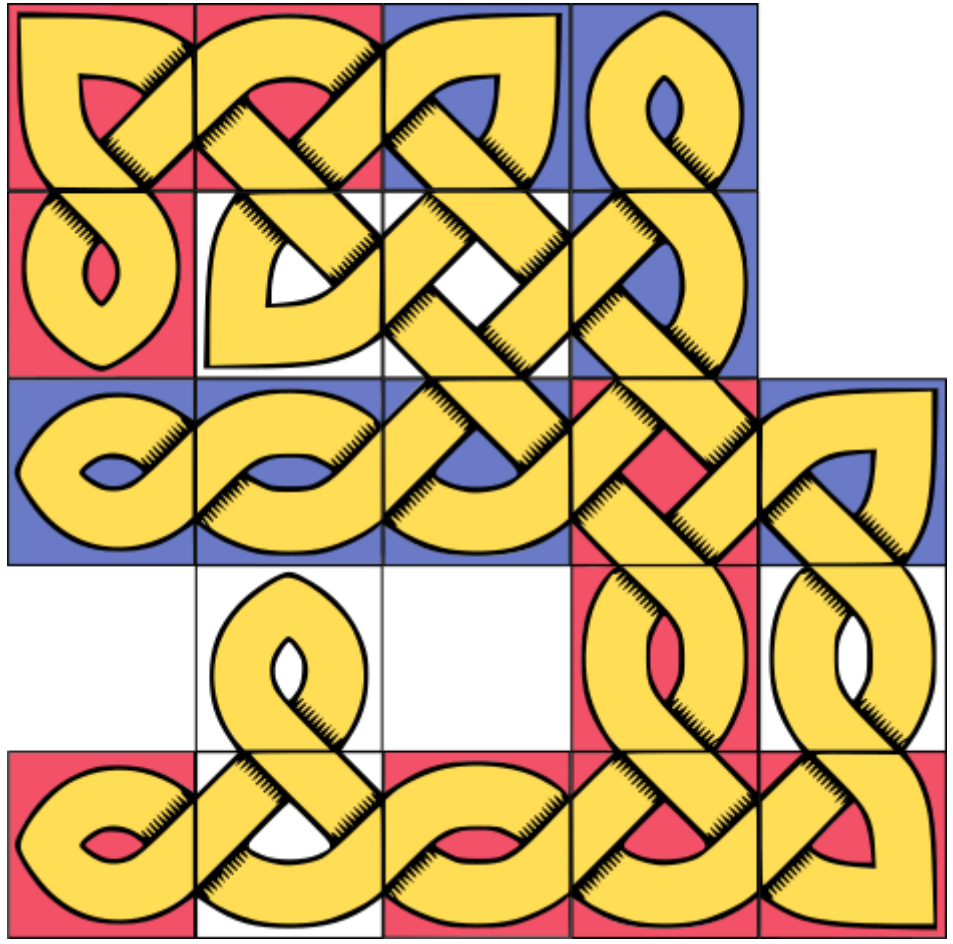
Knot 1: Tie
Knot 2: Red (9)



Celtic! - Scoring

Score

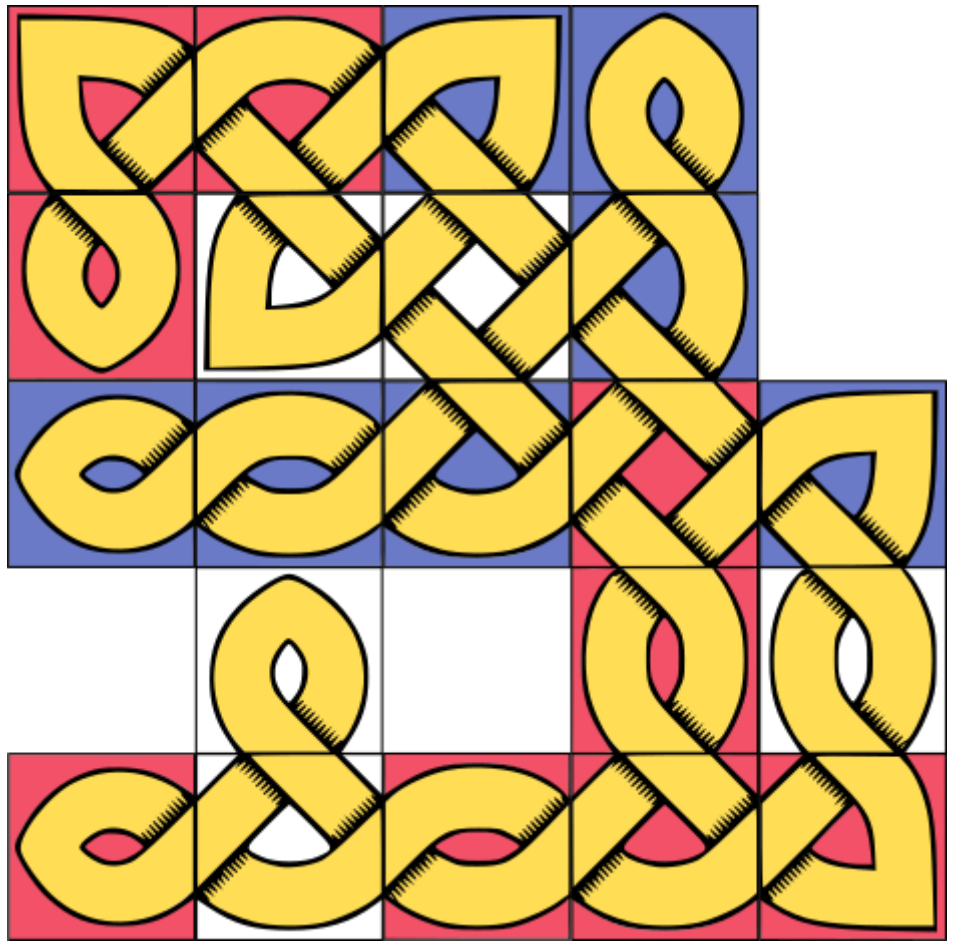
Knot 1: Tie
Knot 2: Red (9)



Celtic! - Scoring

Score

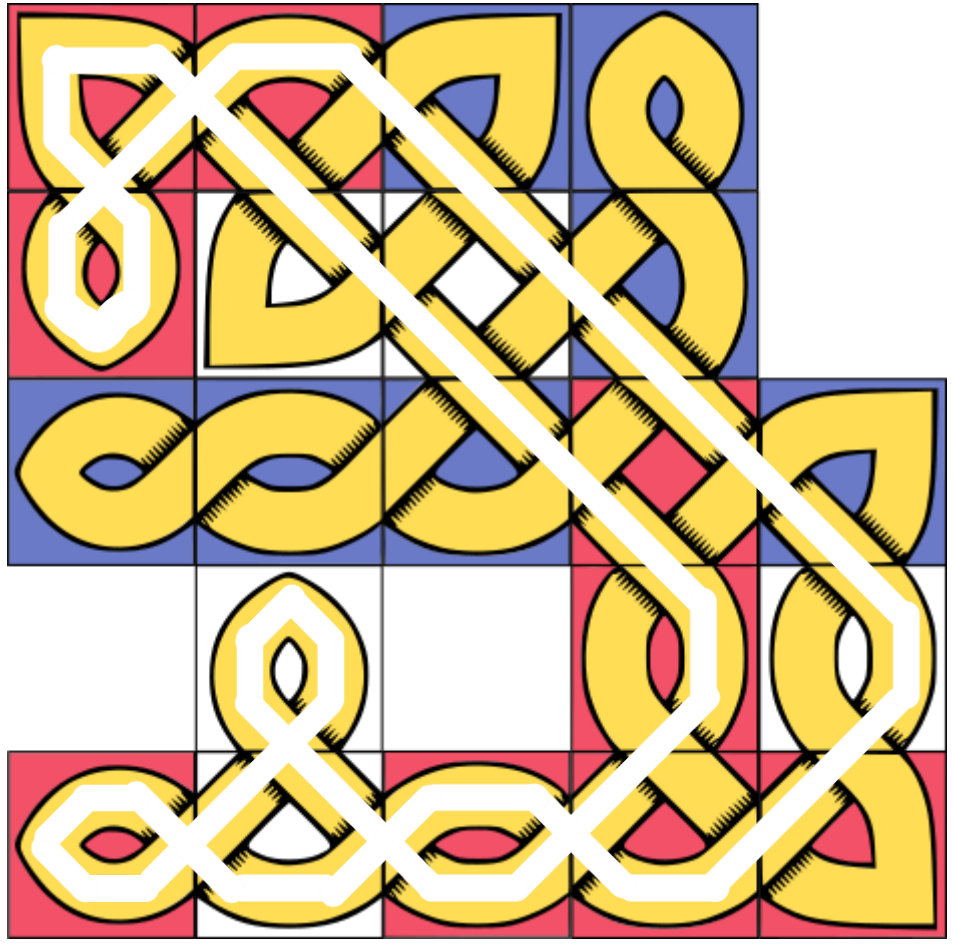
Knot 1: Tie
Knot 2: Red (9)
Knot 3: Blue (5)
Knot 4: Red (4)



Celtic! - Scoring

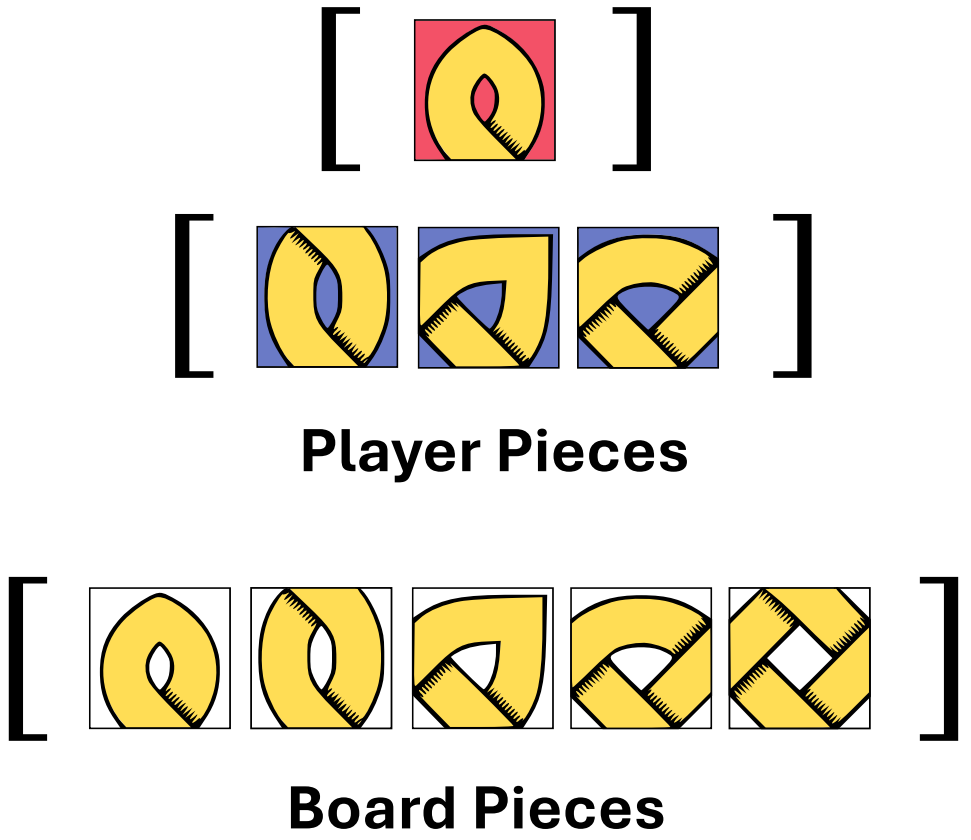
Score

Knot 1: Tie
Knot 2: Red (9)
Knot 3: Blue (5)
Knot 4: Red (4)



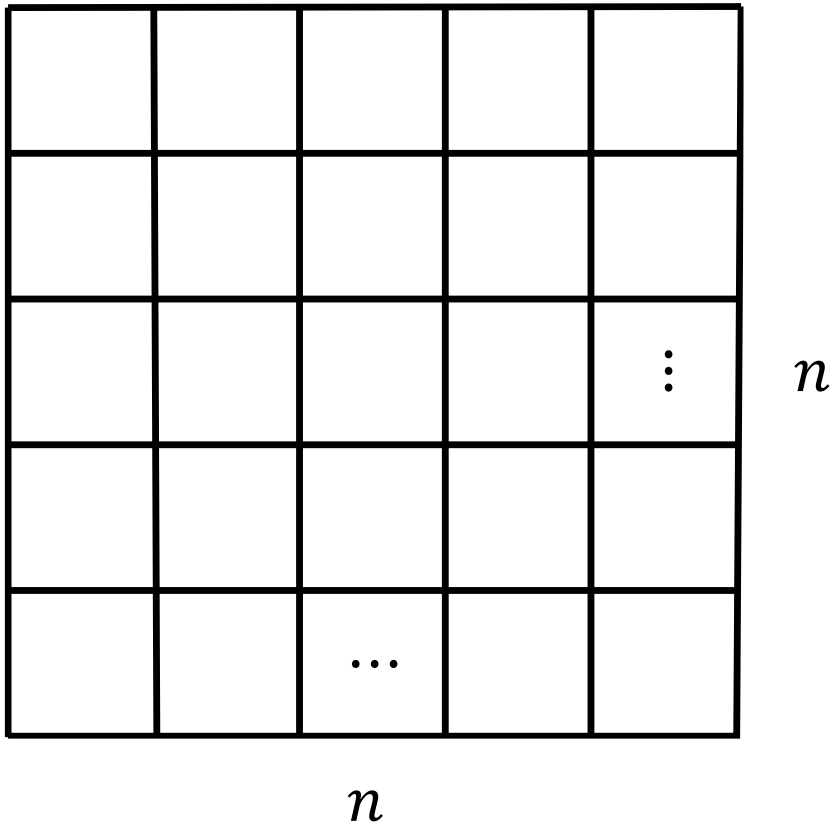
Generalized 2-Player Celtic!

Generalization: 2-Player



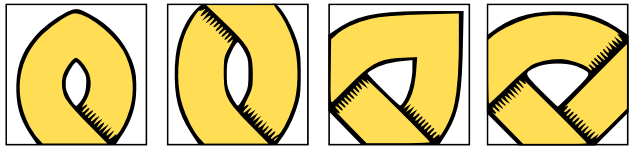
Player Pieces

Board Pieces



Problem: Does the blue player have a winning strategy?

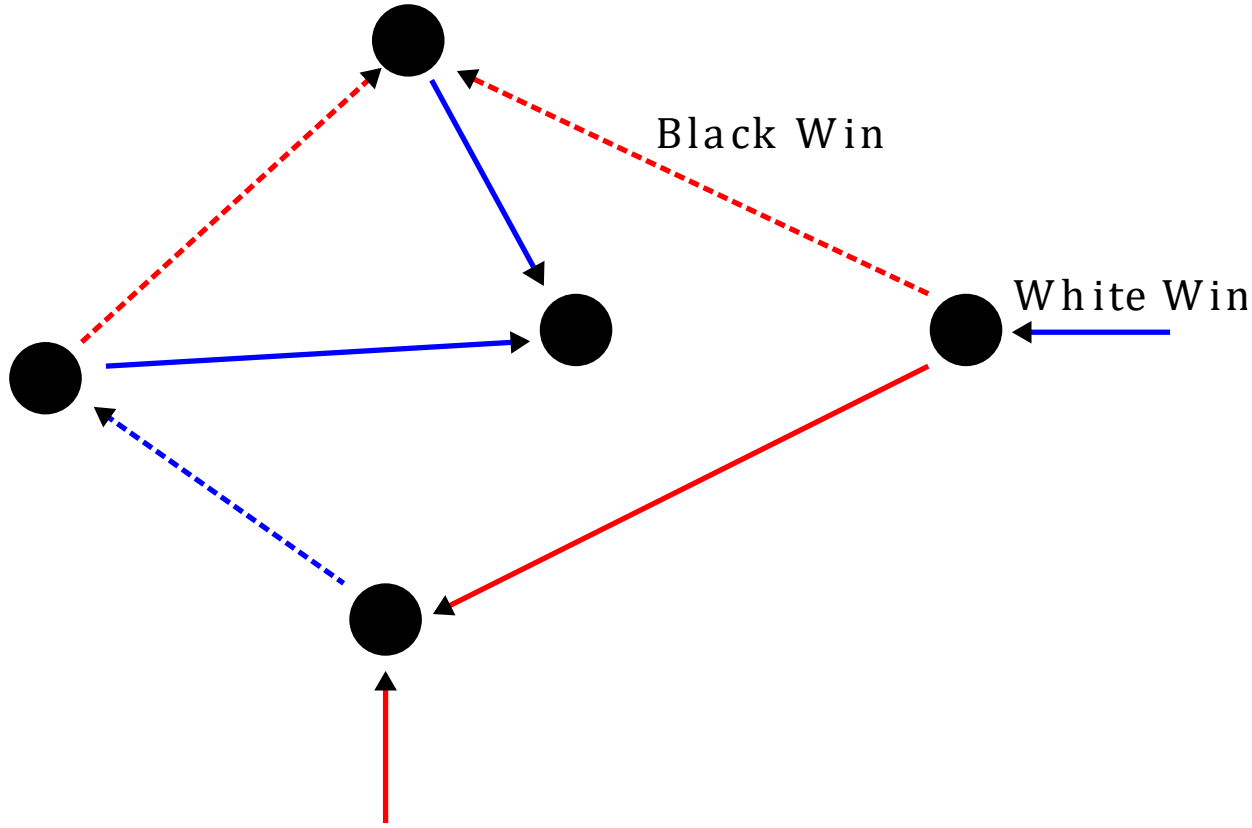
2-Player Celtic!

Player Pieces	Board Pieces	Complexity
		*PSPACE-complete*

Problem: Does the blue player have a winning strategy?

2-Player Celtic!

- Bounded 2-Player Constraint Logic
- Given directed graph with edges of weights 1 or 2
- Edges are partitioned for Black (dotted)/White (solid) players
- Goal is to flip a specific edge



2-Player Celtic!

- Bounded 2-Player Constraint Logic
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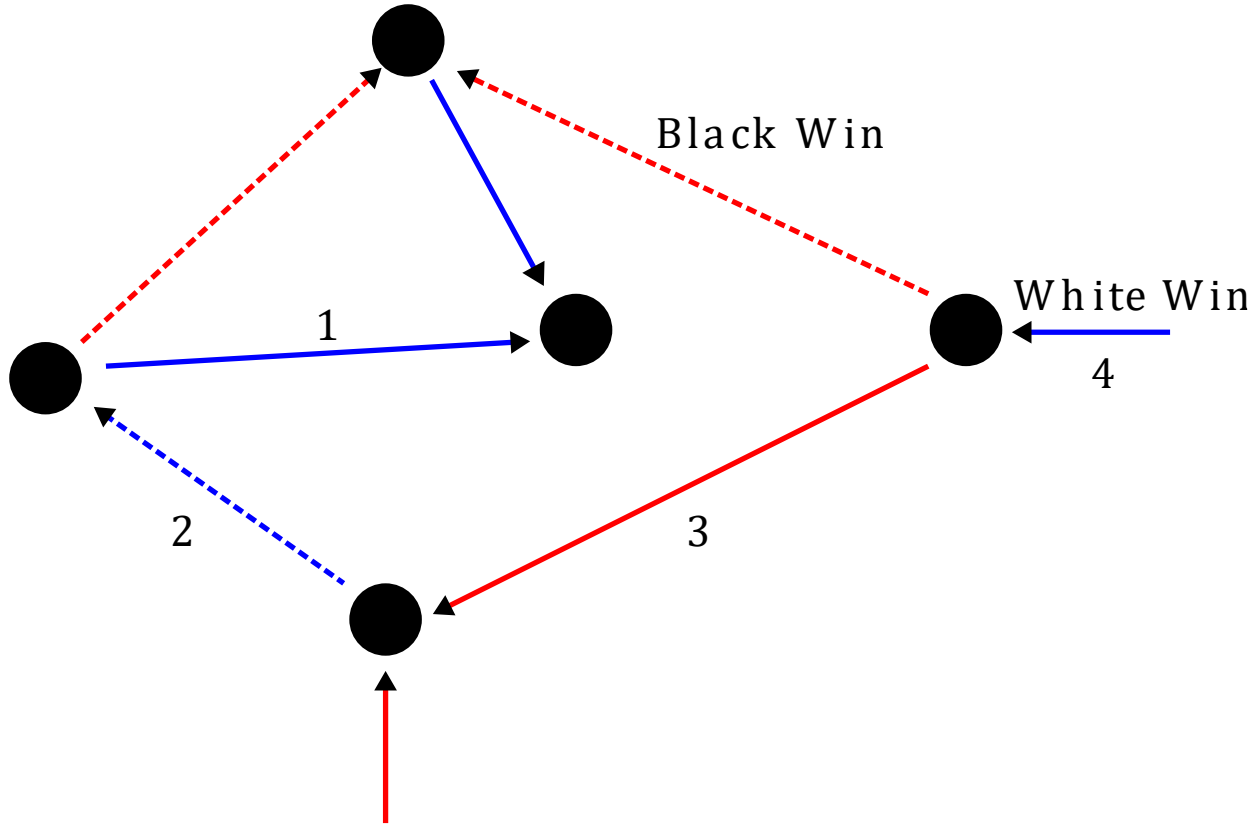
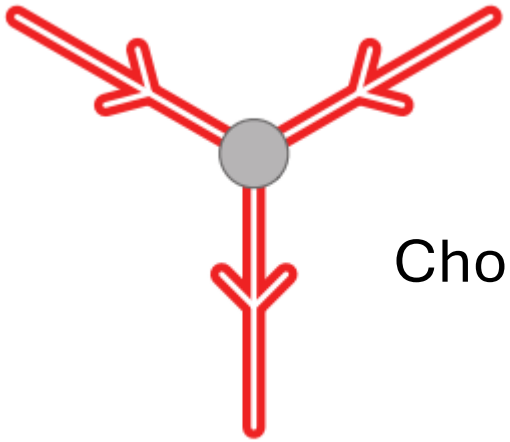
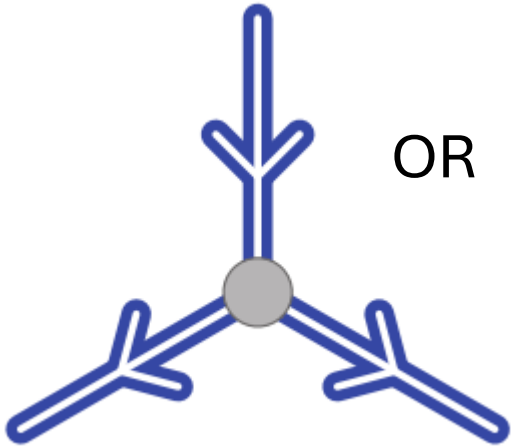


2-Player Celtic!

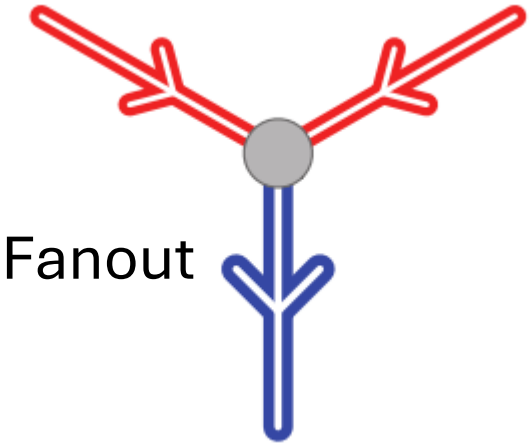


Choice

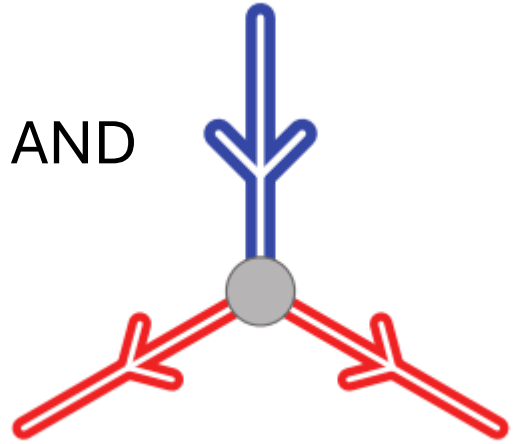


OR

Variable

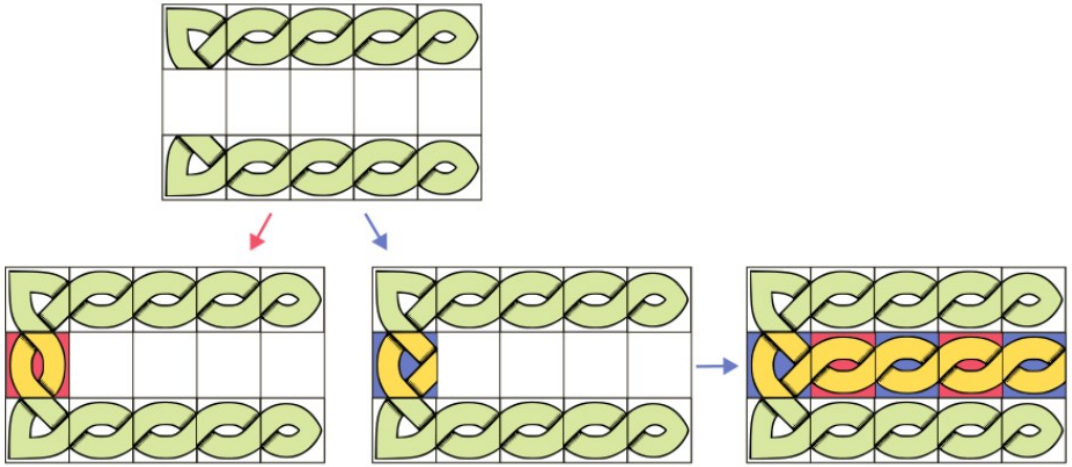


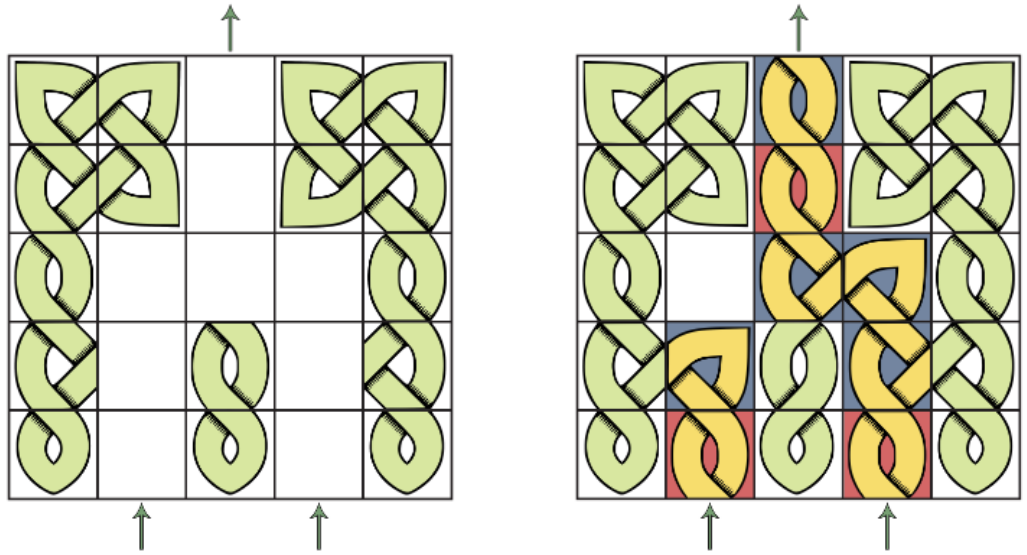
Fanout

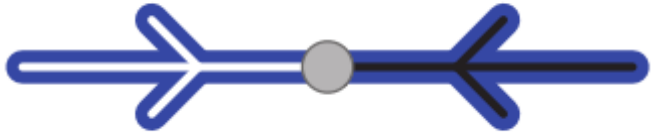


AND

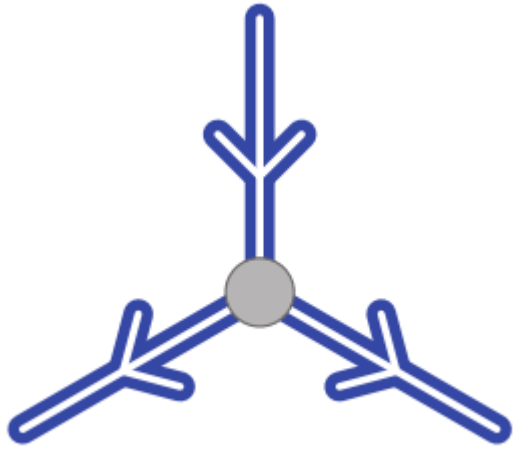
2-Player Celtic!





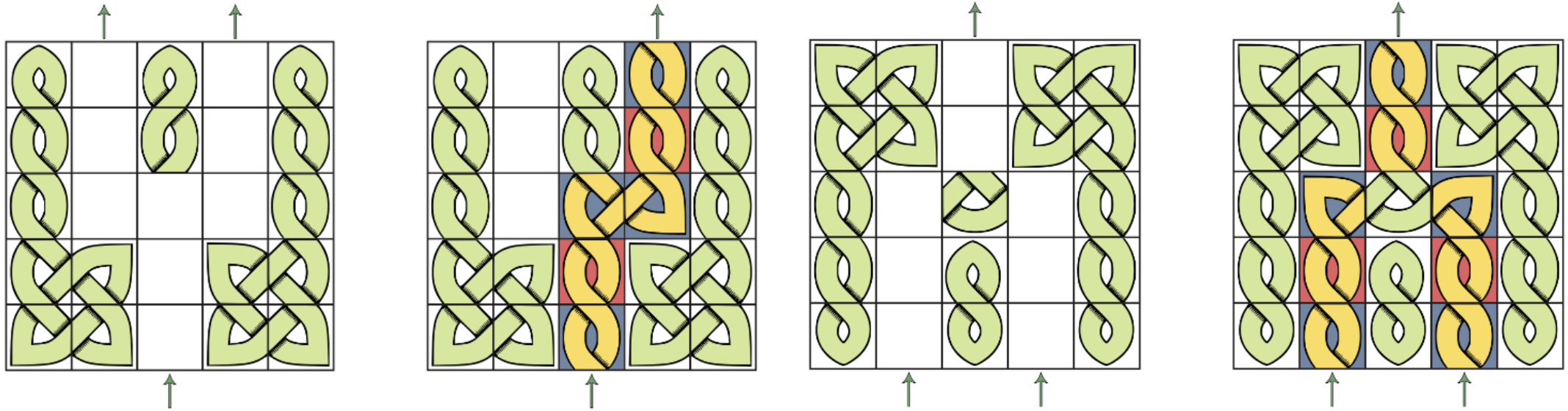


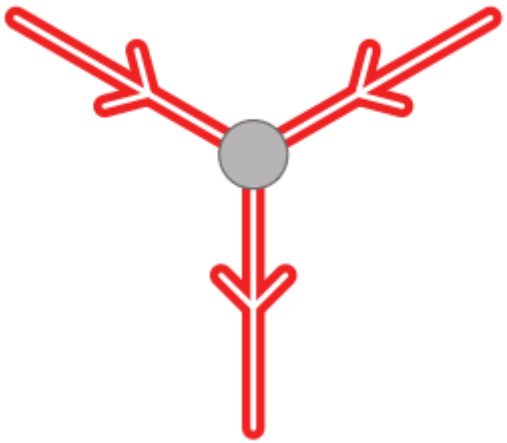
Variable



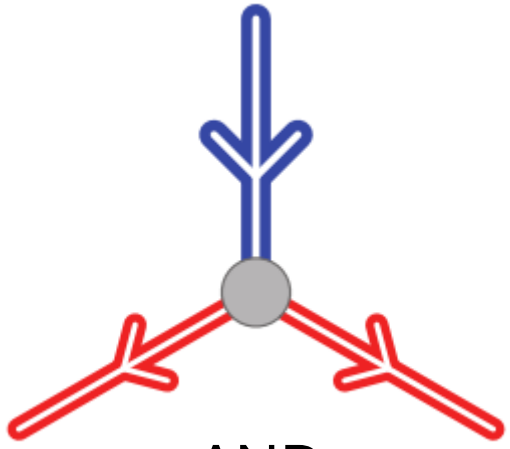
OR

2-Player Celtic!



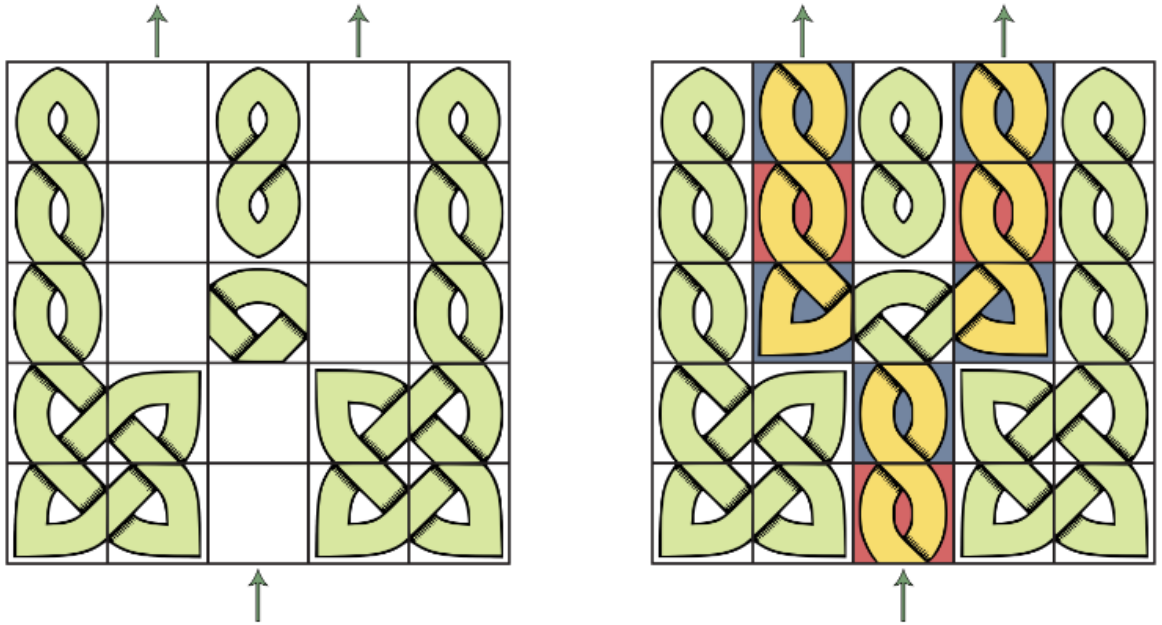


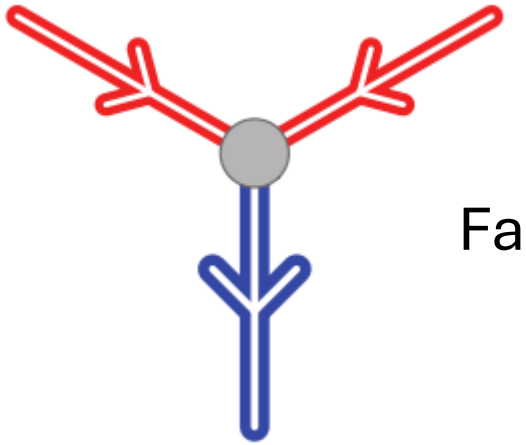
Choice



AND

2-Player Celtic!

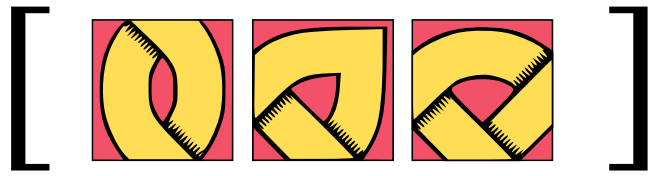




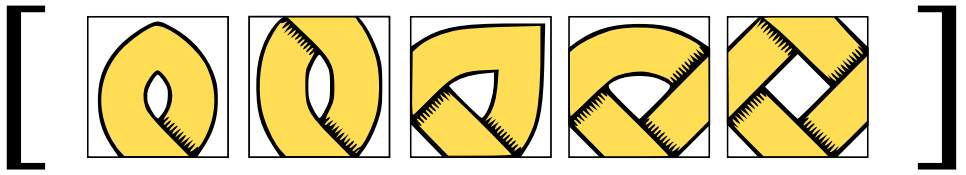
Fanout

Generalized 1-Player Celtic!

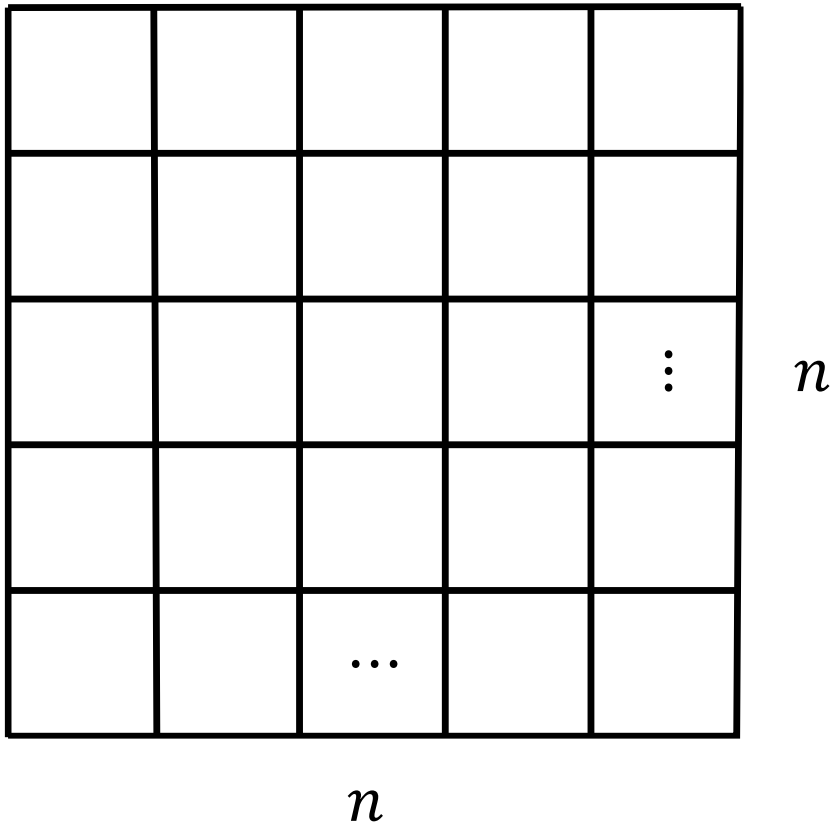
Generalization: 1-Player



Player Pieces



Board Pieces



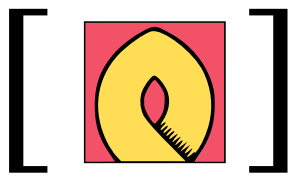
Problem: Can the player build a knot of size $\geq L$ given k pieces?

1-Player Celtic!: The “Easy” problems

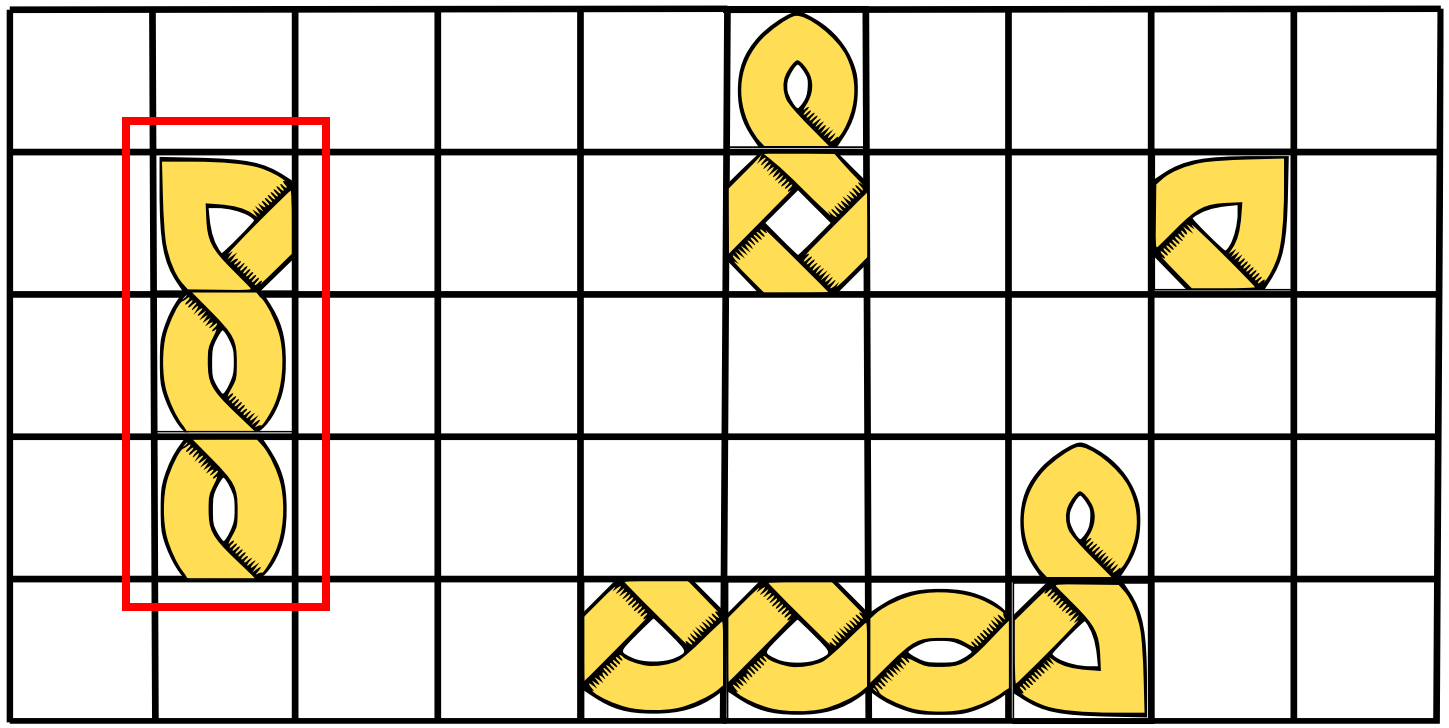
Player Pieces	Board Pieces	Complexity
		$O(n^2)$
		
		

Problem: Can the player build a knot of size $\geq L$ given k pieces?

1-Player Celtic!: The “Easy” problems

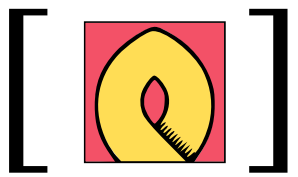


Player Pieces

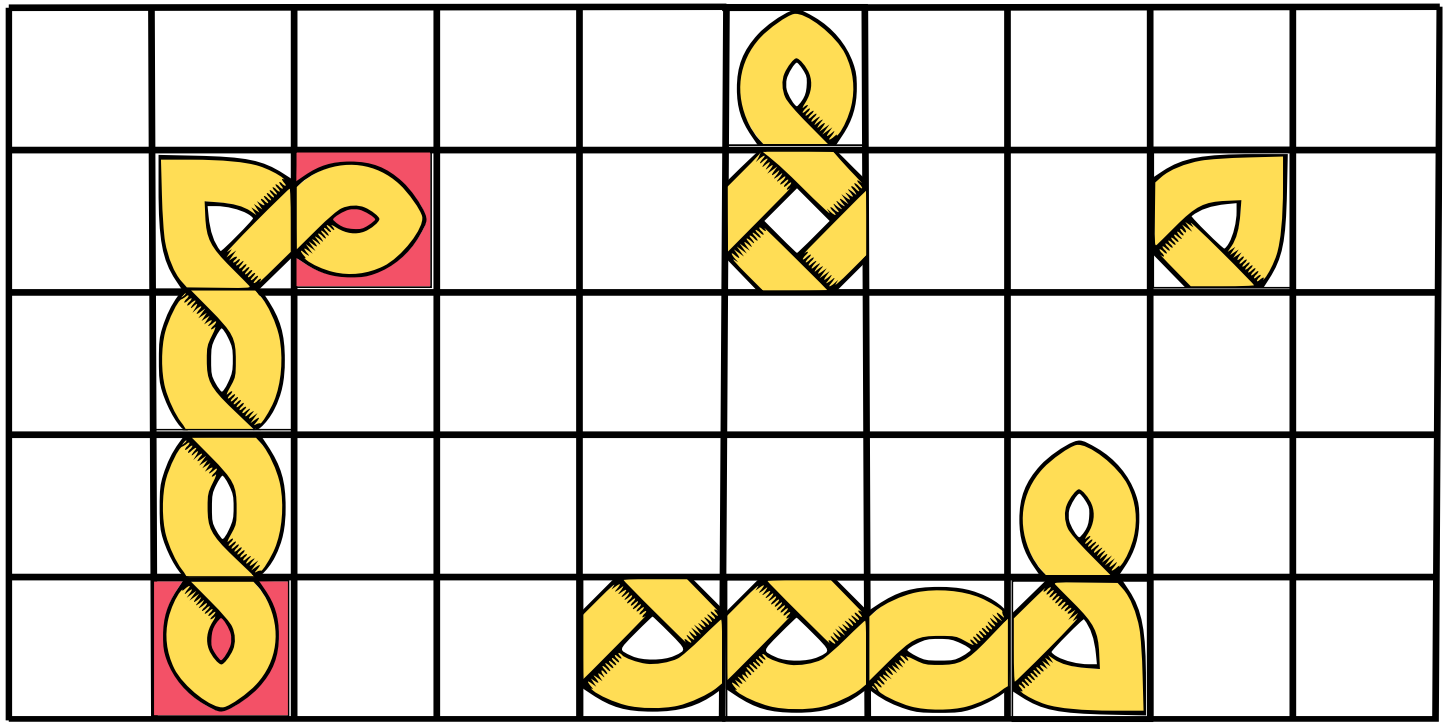


Given: $L = 6,\ k = 4$

1-Player Celtic!: The “Easy” problems



Player Pieces



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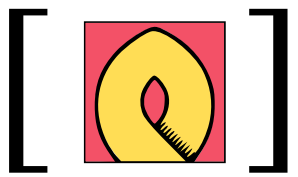
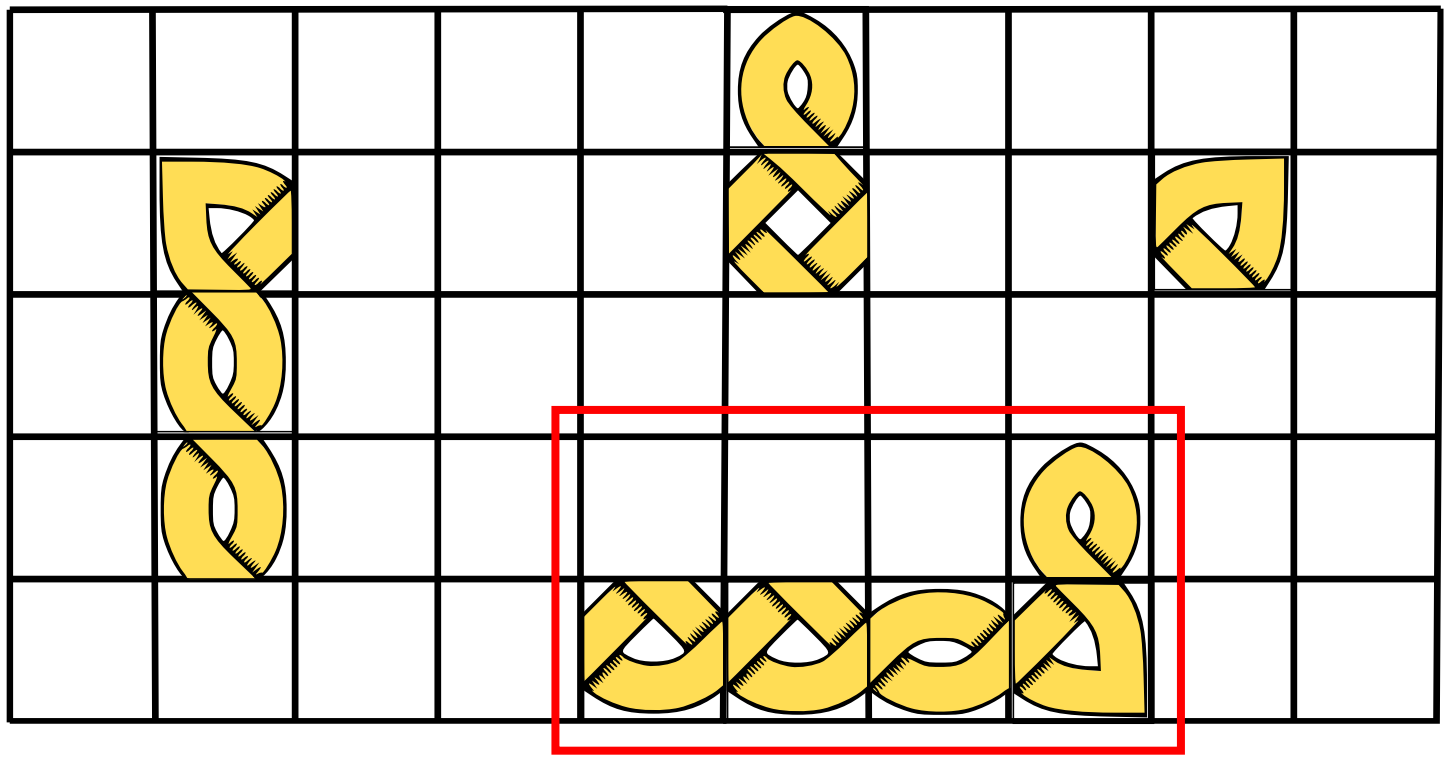
1-Player Celtic!: The “Easy” problems



Player Pieces



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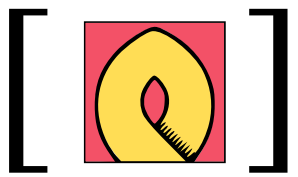
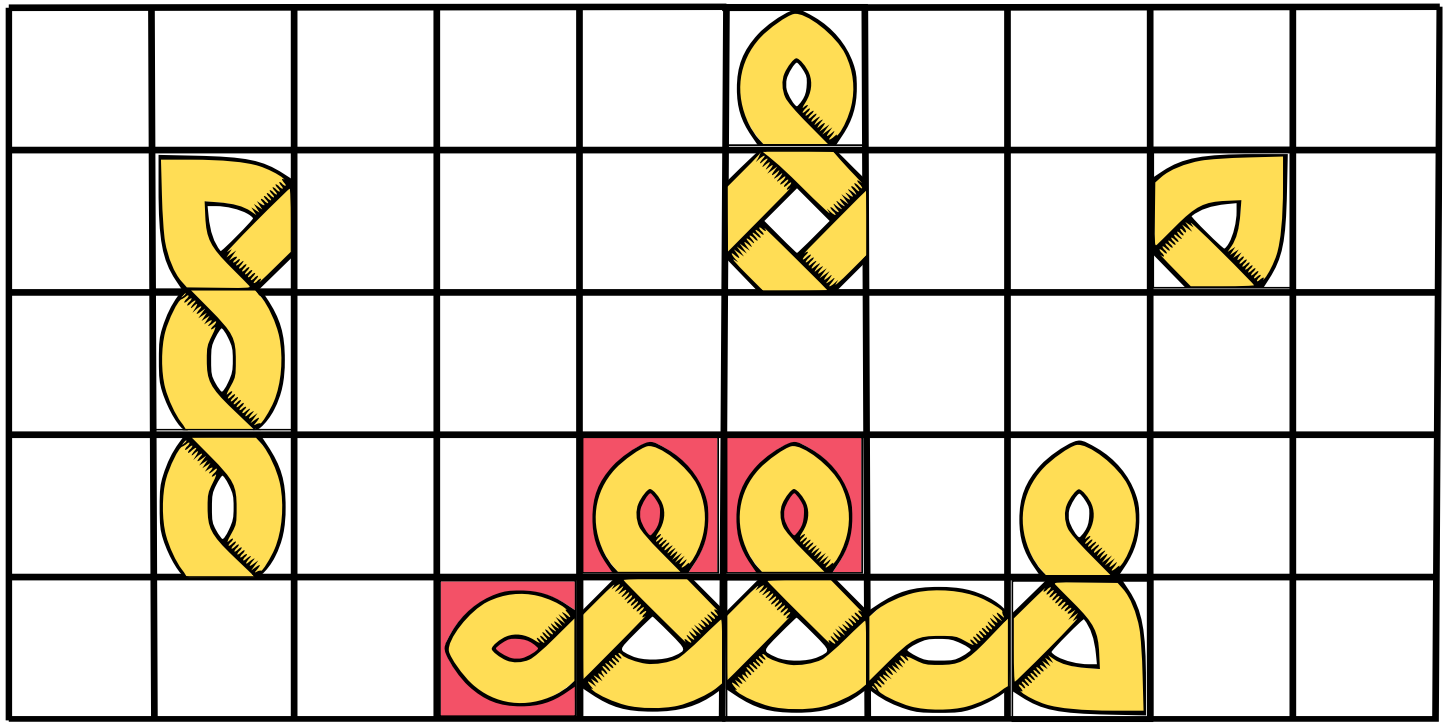


Player Pieces



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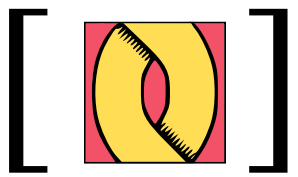
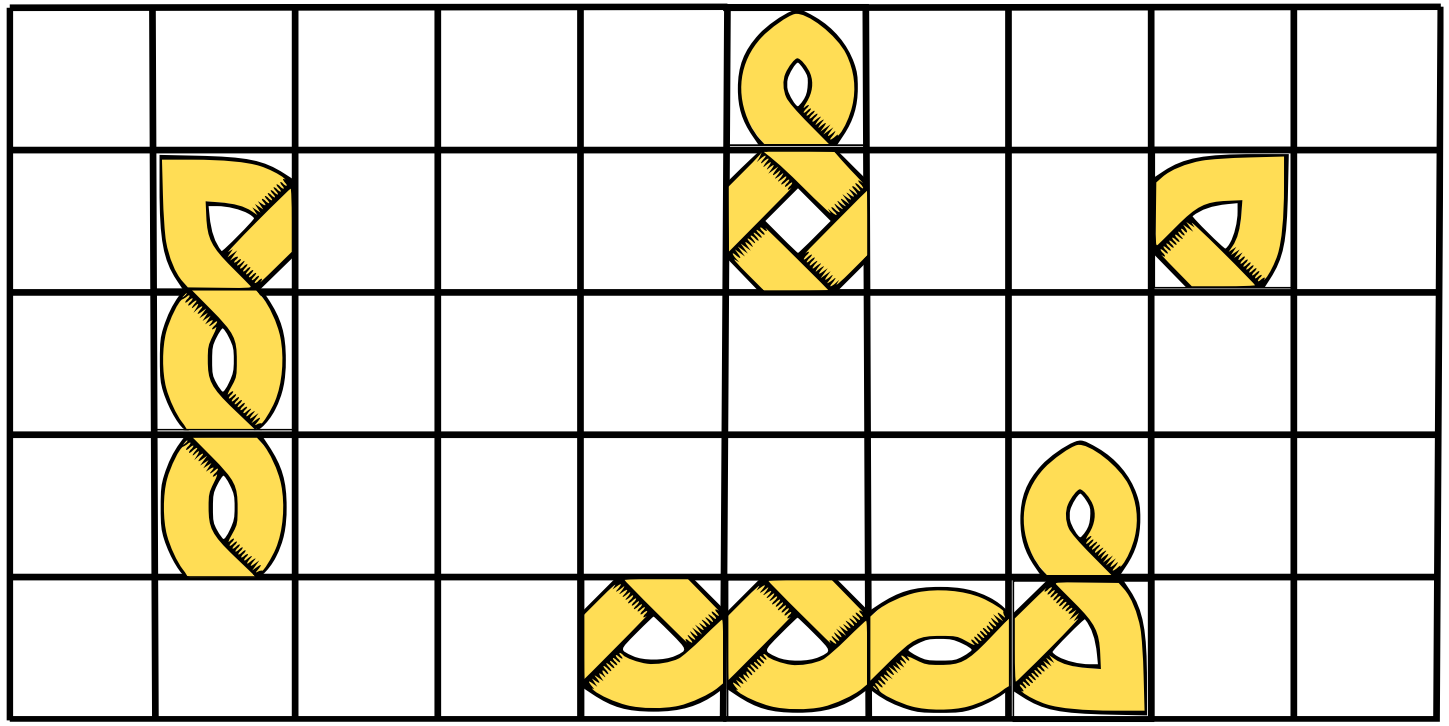
1-Player Celtic!: The “Easy” problems

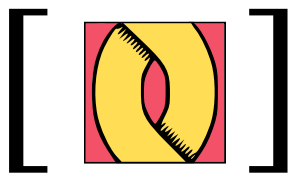


Player Pieces

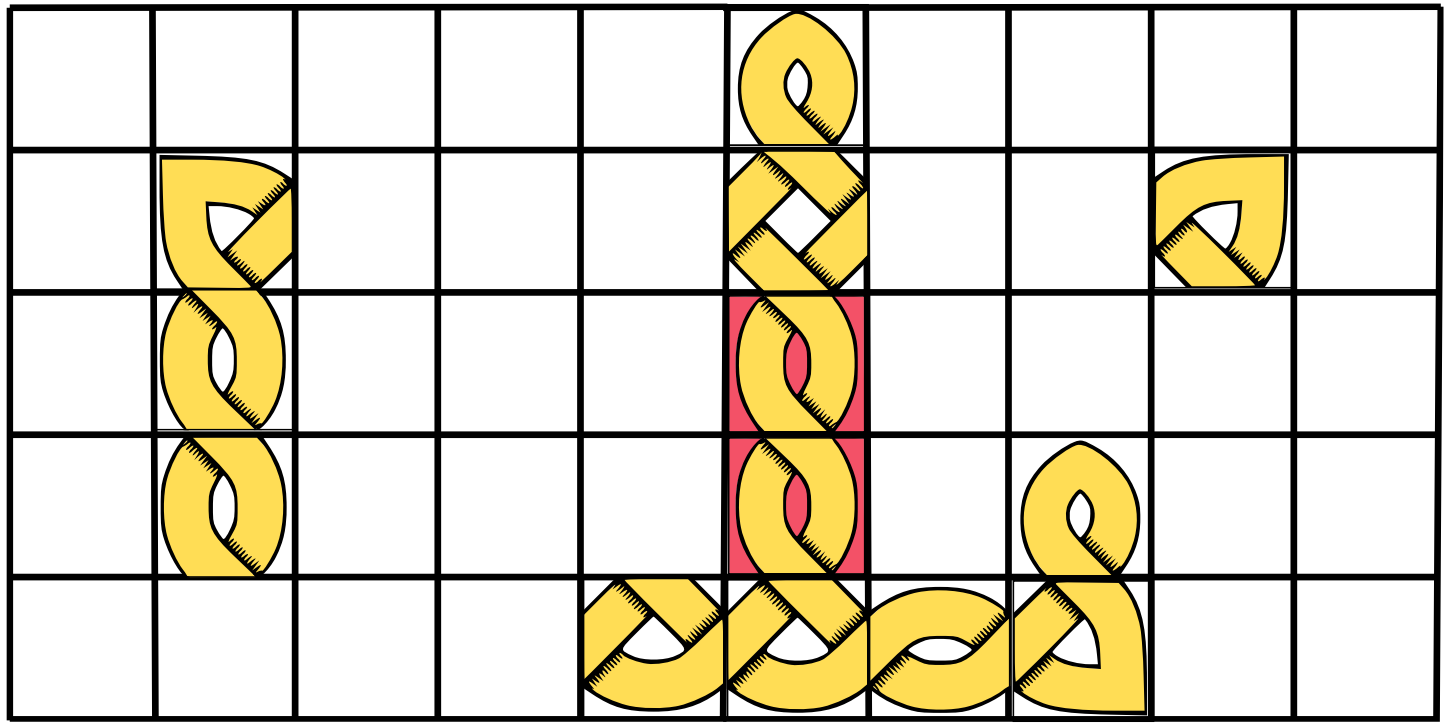


Given: $L = 6,\ k = 4$

1-Player Celtic!: The “Easy” problems

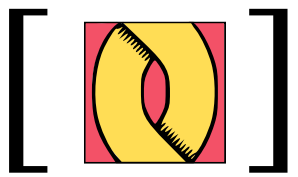


Player Pieces

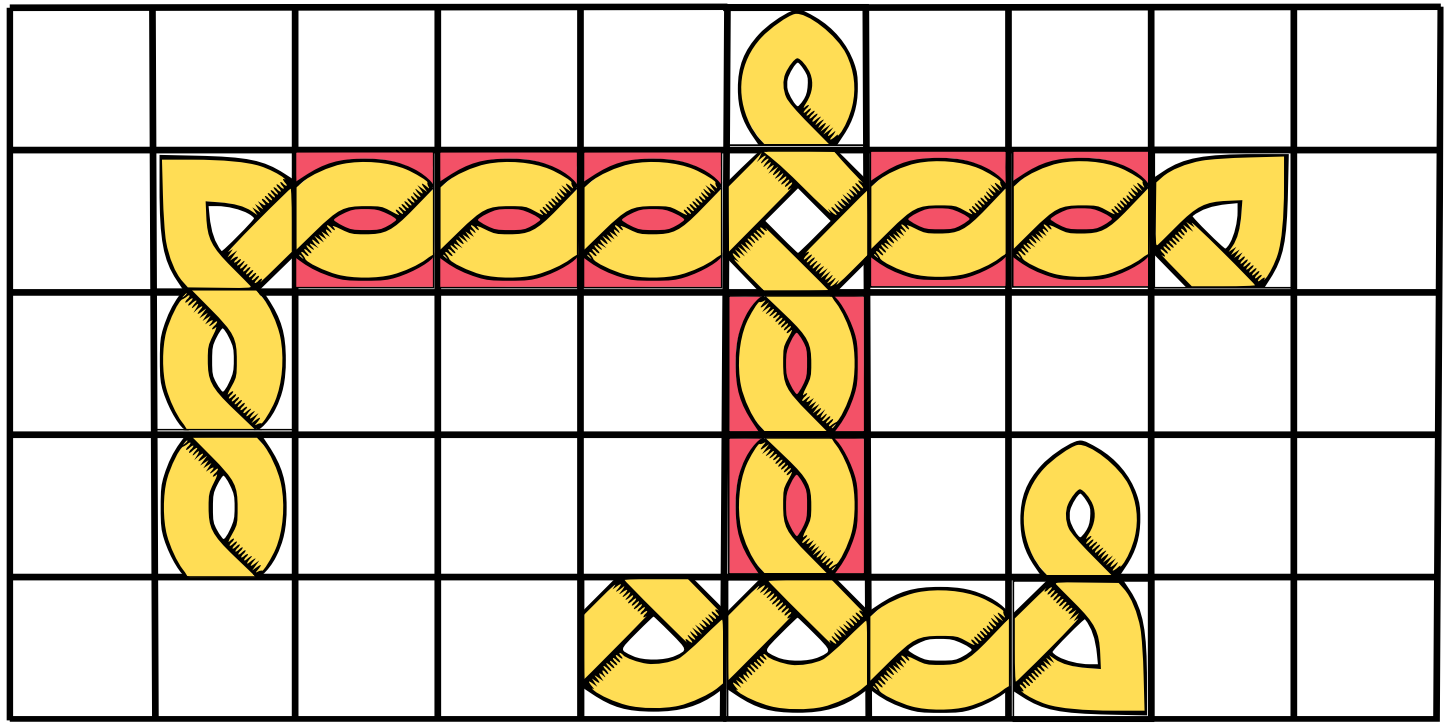


Given: $L = 6,\ k = 4$

1-Player Celtic!: The “Easy” problems

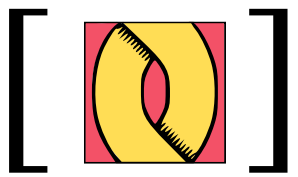


Player Pieces

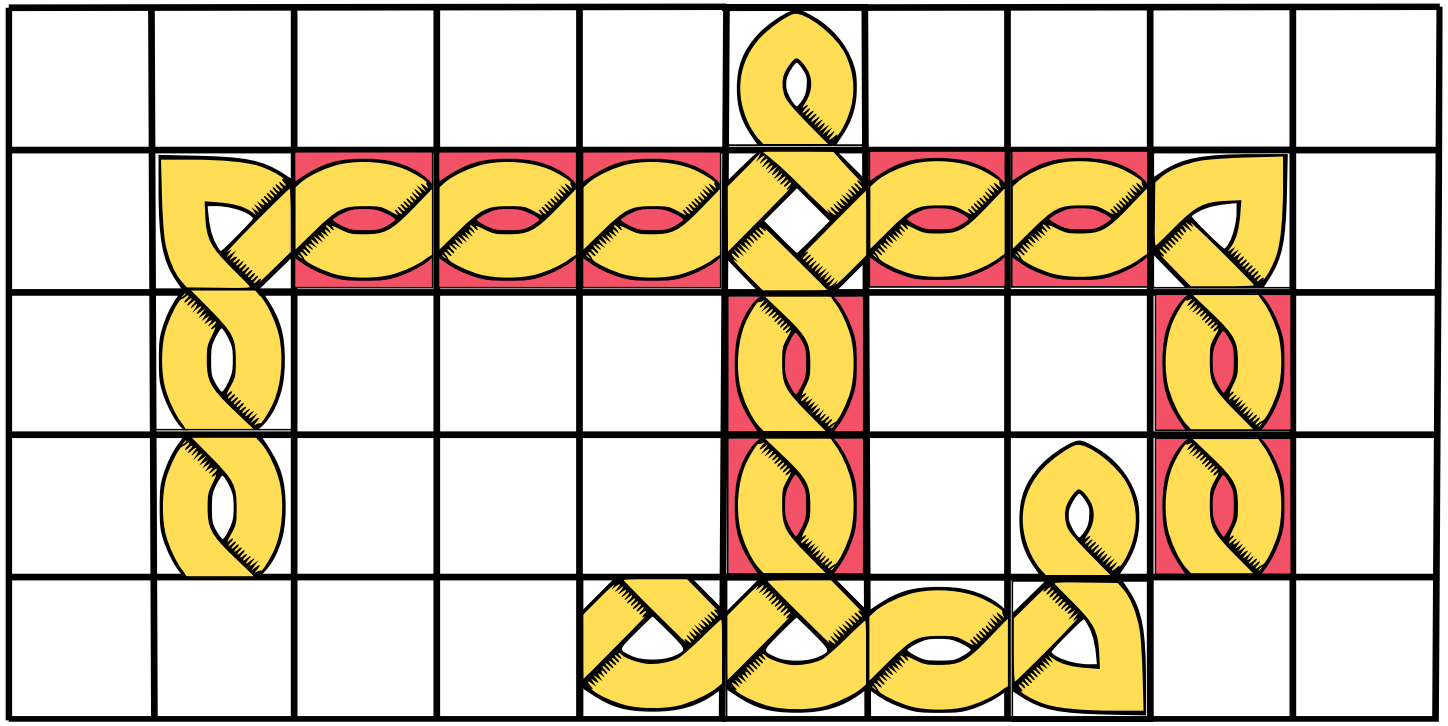


Given: $L = 6,\ k = 4$

1-Player Celtic!: The “Easy” problems

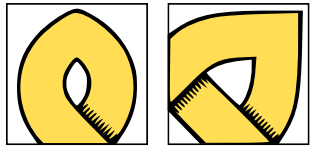
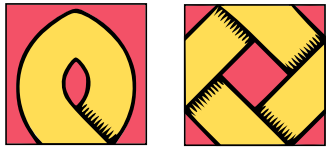


Player Pieces



Given: $L = 6,\ k = 4$

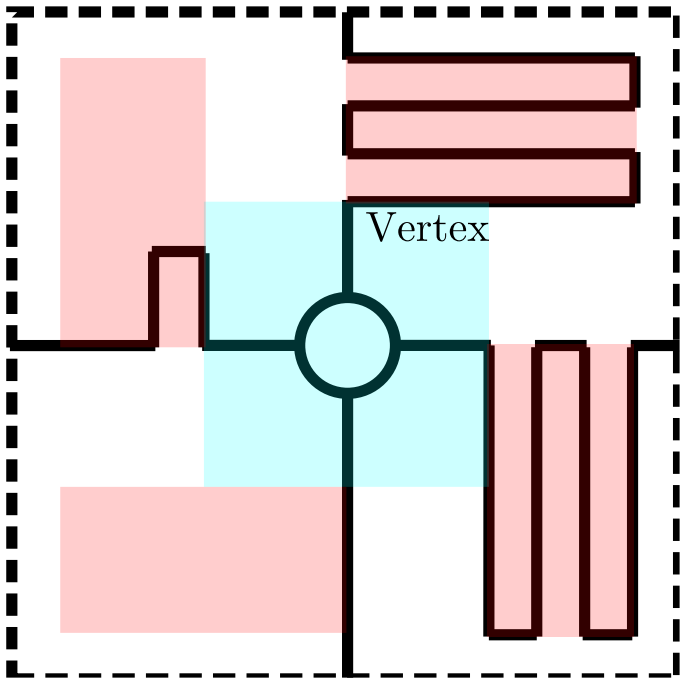
1-Player Celtic!: The “Hard” problems

Player Pieces	Board Pieces	Complexity
		*NP-complete*
		
		

Problem: Can the player build a knot of size $\geq L$ given k pieces?

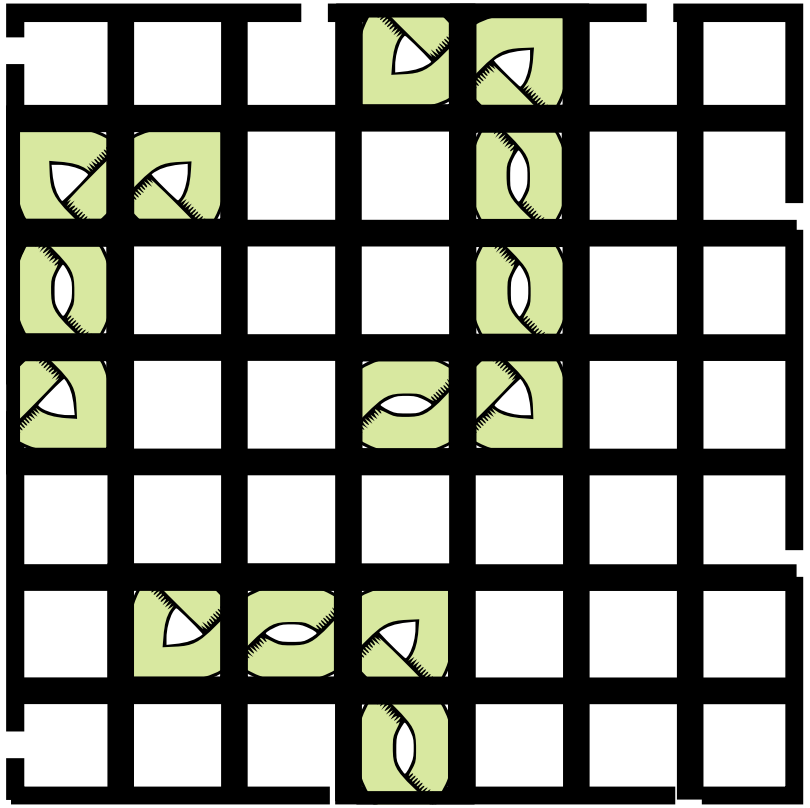
1-Player Celtic!: The “Hard” problems

- Planar, directed Hamiltonian Cycle with max degree 3
- Transform graph into equivalent rectilinear embedding with area $(|V| + 1) \times (|V| + 1)$
- Scale new graph by $O(|V|)$
- Edges are pumped to be roughly the same



1-Player Celtic!: The “Hard” problems

- Center piece represents the “vertex”
- Incoming edge(s) are moved to one side
- Outgoing edge(s) are moved to the other side
- The “choice” is made by the played pieces



1-Player Celtic!: The “Hard” problems

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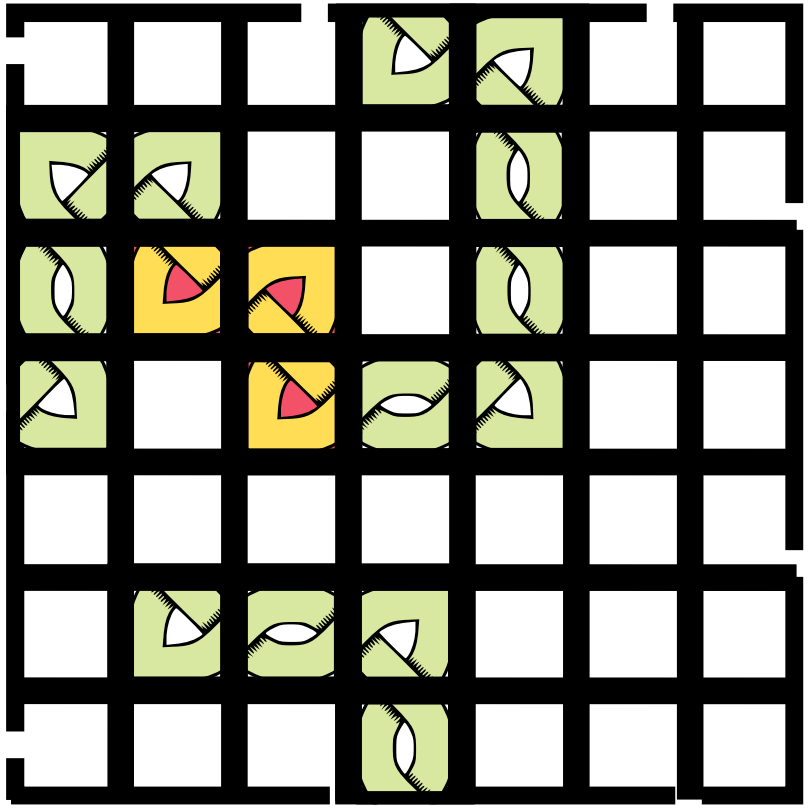
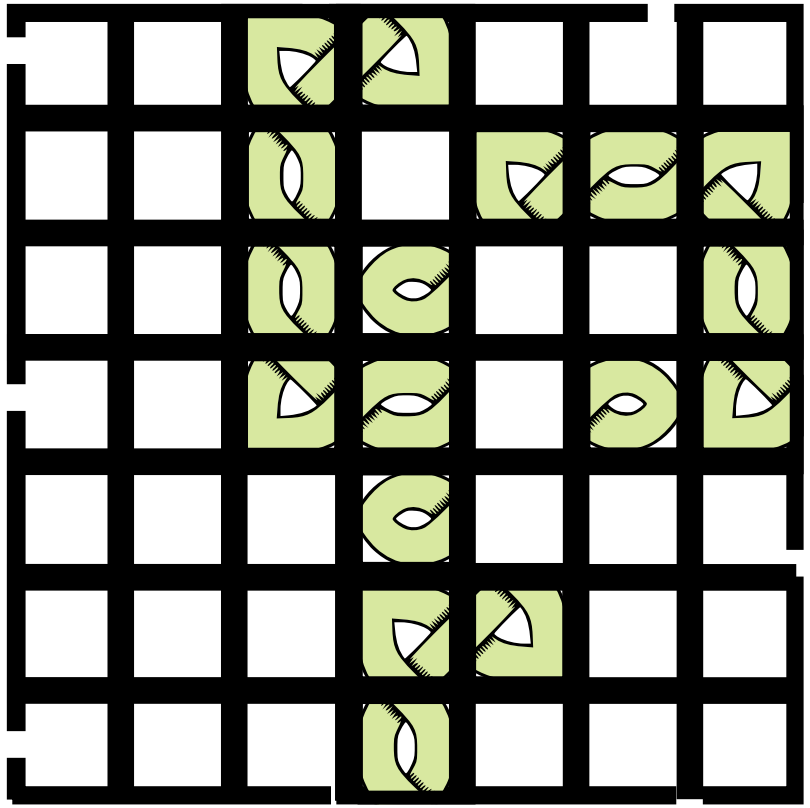
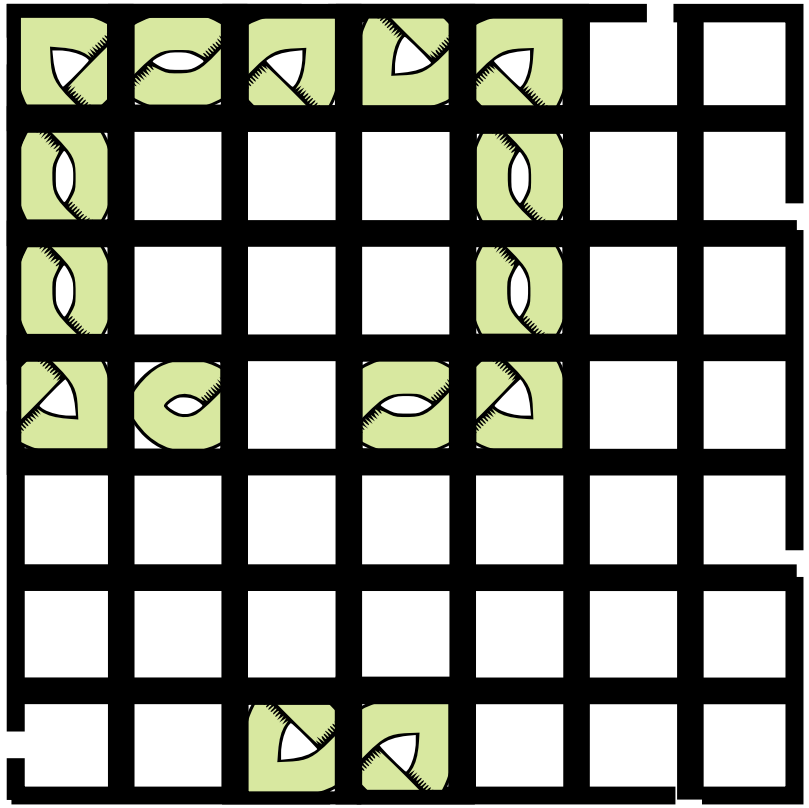
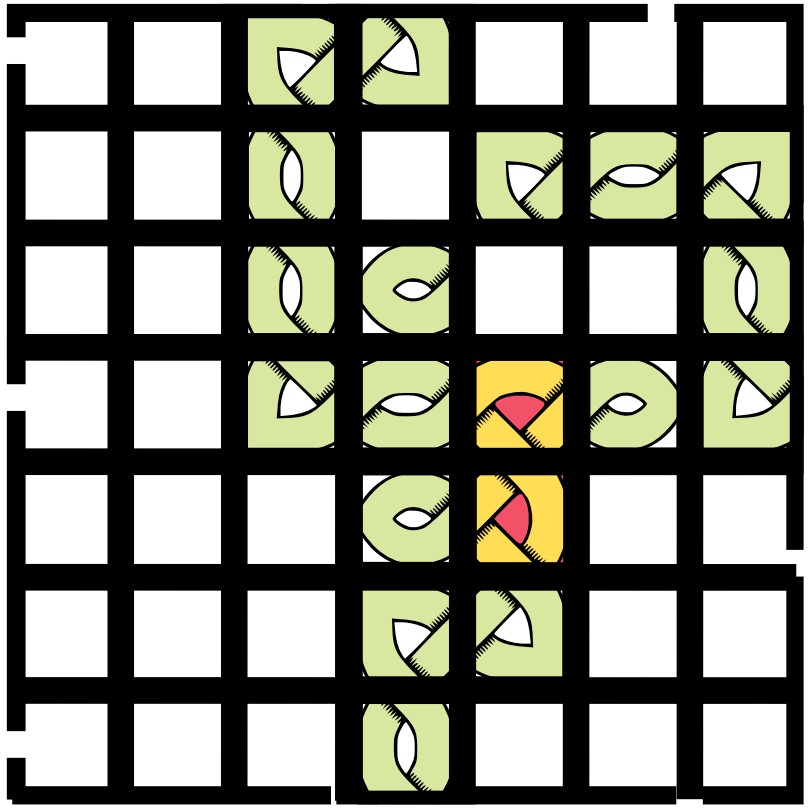


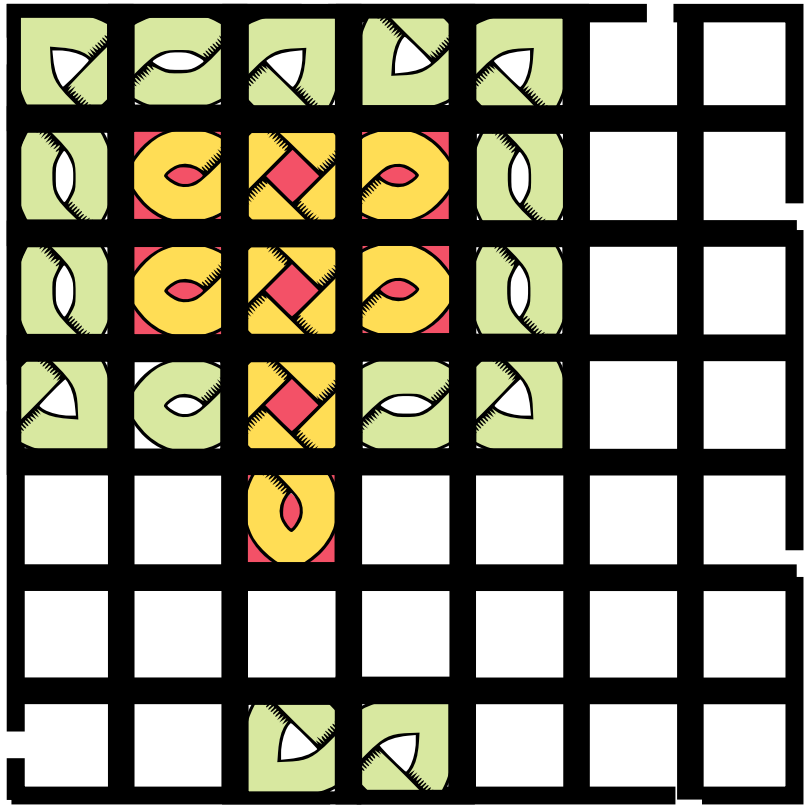
1-Player Celtic!: The “Hard” problems





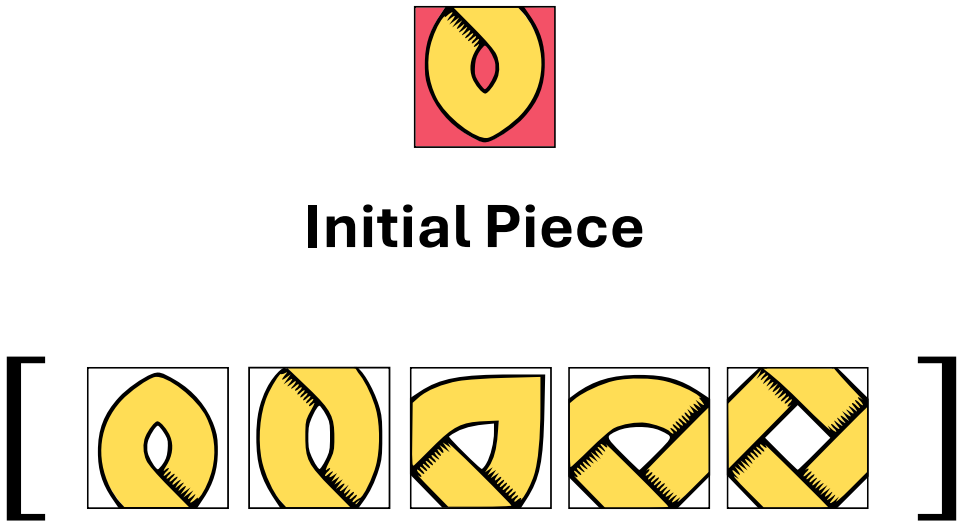
1-Player Celtic!: The “Hard” problems





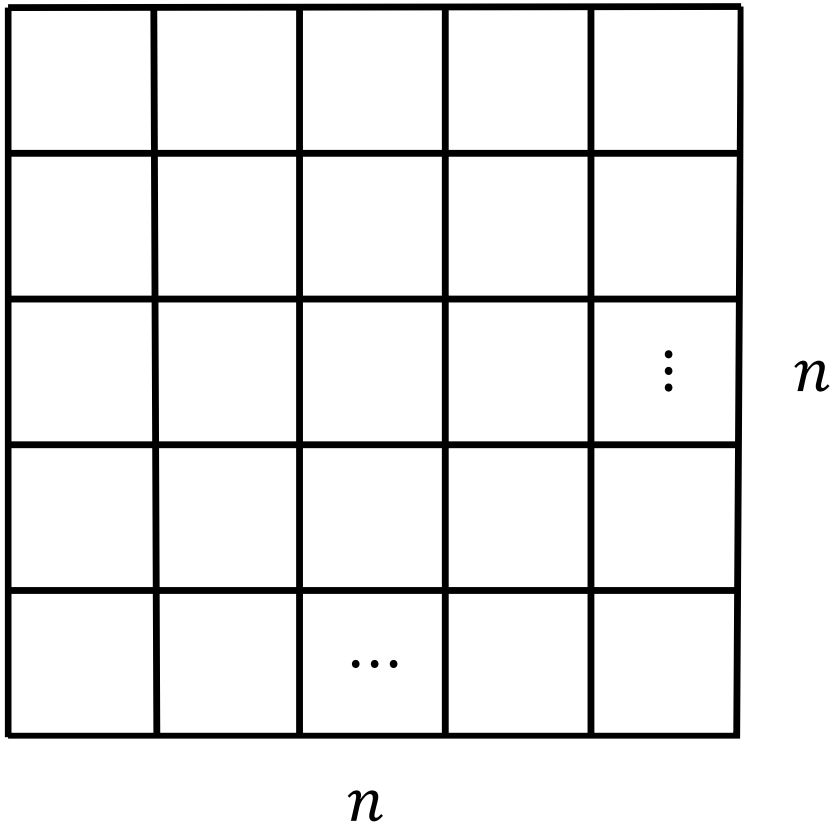
Generalized 0-Player Celtic!

Generalization: 0-Player



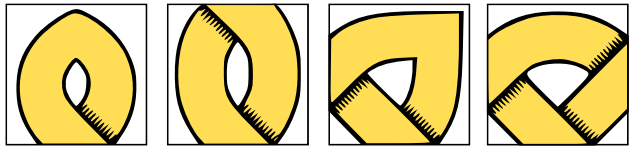
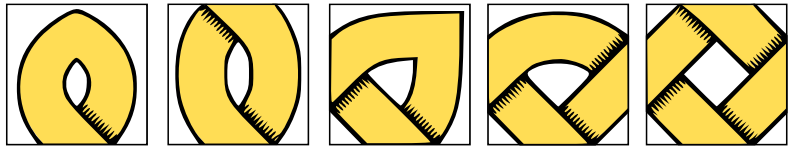
Initial Piece

Board Pieces



Problem: Does the simulation build a knot of size $\geq L$?

0-Player Celtic!

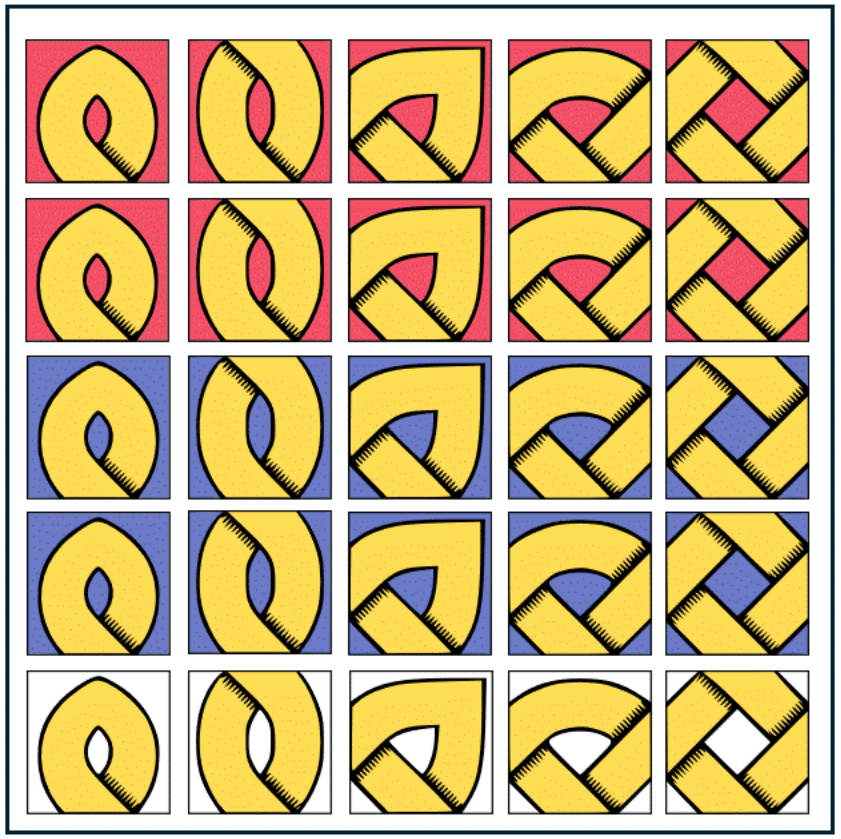
Player Pieces	Board Pieces	Complexity
		*P-complete*

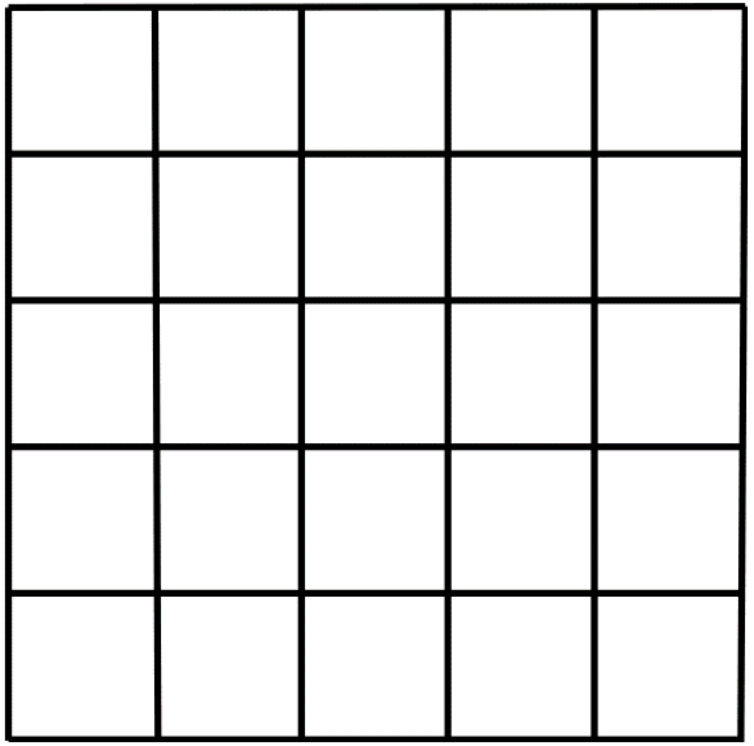
Problem: Does the simulation build a knot of size $\geq L$?

Conclusion

- Some piece combinations missing for Generalized 1-Player Celtic!
- More balanced pieces for Generalized 2-Player Celtic!
- FPT, approximations for maximally sized knots
- Is Celtic! a 1st player win, 2nd player win, or a draw?
- Pattern based complexity of the knots.
- Consider > genus-1 knot assembly.

Tile-based Knot Assembly with Celtic!





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